Assessing Patent Strength Using Data-Driven Inputs: Characteristics Of Patents And Patent Owners That Drive Success In *Inter Partes* Review

By Christopher K. Larus, Miles A. Finn, Shelley R. Gilliss, Jared A. Berry and Congnan Zhan

Introduction

The environment for patent licensing and enforcement is rapidly transforming. *Inter partes* review and other post-grant proceedings under the America Invents Act, changes to legal standards governing patent eligibility, and increasingly stringent review of patent damage awards have all combined to alter substantially the risk profile associated with patent licensing and enforcement. In this highly dynamic environment, patent holders, capital investors, and potential licensees alike increasingly look for data-driven quantitative inputs to evaluate patent-related risk.

This paper details our analysis of objective, publicly available patent data to evaluate the relative strength of patent portfolios. This type of data-driven analysis can provide valuable inputs to parties selecting patents for potential licensing, evaluating potential investments in patent-related enterprises, and evaluating potential risks associated with patent licensing and enforcement.

Evaluating the overall strength of a patent portfolio is a highly fact-intensive undertaking, and approaches to evaluating patent strength can vary widely depending upon the purpose of the evaluation. Fundamentally, however, patents grant legal rights, and any measure of patent strength must consider the ability of the patent holder to withstand a legal challenge to its patent rights. Thus, a key indicator of patent strength is the relative probability that a patent will survive a challenge in an *Inter Partes* Review (IPR) proceeding, compared to similar patents. By "similar patents," as will become clear, we mean patents that share publicly available, machine-extractable characteristic(s) with the subject patent. This definition includes characteristics of patents' assignees.

A complete analysis of patent strength should, of course, include other important inputs. These include, for example, the ability to withstand invalidity challenges that cannot be raised in an IPR, and whether sufficiently valuable infringement can be proven. These other inputs are perhaps amenable to the type of quantitative data-driven analysis reflected in this paper, but they are more challenging to analyze. But given the substantial growth in the number of IPR challenges, the impact of these proceedings on patent enforcement, and our ability to quantify IPR data, the relative ability of a patent to survive an IPR challenge is perhaps the key quantifiable input to the evaluation of risk in patent licensing and enforcement. Moreover, even where a portfolio owner has no interest in enforcing its portfolio, potential licensees will likely evaluate the prospect of a successful IPR challenge when considering alternatives to licensing. Accordingly, we argue that the techniques described below form the basis for portfolio valuation that every portfolio owner should consider using.

Literature Review

Others have analyzed patent strength using techniques similar to ours. For example, in 2015, Allison, Lemley, and Schwartz analyzed district court outcomes for 945 cases filed from 2008-2009.[1] They categorized the litigated technology as belonging to one of

1. John R. Allison, Mark A. Lemley, and David L. Schwartz, "Our Divided Patent System," *The University of Chicago Law Review* Vol. 82, No. 3 (Summer 2015):1073-1154.

2. Scott D. Bass and Lukasz A. Kurgan. "Discovery of Factors Influencing Patent Value Based on Machine Learning in Patents in the Field of Nanotechnology." *Scientometrics* 82, No. 2 (2010): 217-241; Jungpyo Lee and So Young Soh, "What Makes the First Forward Citation of a Patent Occur Earlier?" *Scientometrics*, (2017): 1588, *https://doi.org/10.1007/s11192-017-2480-1.*

■ Christopher K. Larus,
Robins Kaplan LLP,
Partner; Chair,
Minneapolis IP and Technology Litigation Group
Minneapolis, Minnesota, USA
E-mail: clarus@robinsKaplan.com

■ Miles A. Finn Ph.D.,
Robins Kaplan LLP,
Associate,
New York, New York, USA
E-mail: mfinn@robinsKaplan.com

■ Shelley R. Gilliss, Ph.D.,
Robins Kaplan LLP,
Chief Scientist,
Minneapolis, Minnesota, USA
E-mail: SGilliss@RobinsKaplan.com

■ Jared A. Berry, Esq.,
Robins Kaplan LLP,
Senior Consultant, Intellectual Property Economics,
Minneapolis, Minnesota, USA
E-mail: JBerry@RobinsKaplan.com

■ Congnan Zhan, Ph.D.,
Robins Kaplan LLP,
Economic Consultant,
Competition Economics,
Minneapolis, Minnesota, USA
E-mail: CZhan@RobinsKaplan.com

six technology areas—mechanical, electrical, chemistry, biotechnology, software, and optics, as well as to one of 11 industry categories (including computer and other electronics, semiconductor, pharmaceutical, biotechnology, communications, transportation, construction, and energy). In 2010, Bass and Kurgan analyzed factors, including forward citations, in determining patent value.[2]

Our approach to the problem of assessing patent strength differs from these studies in numerous ways. Two differences are most obvious. First, we measure patent strength differently, using a claim's ability to survive an IPR as the key metric. Second, we use a larger dataset restricted to a relatively small number of years (2012-2016).

Materials and Methods

Data Sources and Inclusion Criteria

We based our analysis on the text and bibliographic data of the entire population of 3,482 patents involved in 5,561 IPR proceedings for which a petition was filed from the first availability of the IPR process (in September 2012) through December 31, 2016, and also included those for which a Final Written Decision (FWD) was issued on or before December 31, 2016. We obtained that data from the Patent Trial and Appeal Board's (PTAB) End-to-End data base, the PTAB's API, and the IP Data Direct database (IPDD) available from LexisNexis.[3] We excluded design patents from our data set.

How We Characterized IPR Outcomes

Our assessment of patent strength focused on the relative probability that the subject patent would survive the IPR process with at least one challenged claim intact. Accordingly, we looked at the outcomes of various events in IPR proceedings in three ways: 1. a statistical analysis of Institution Decisions (ID); 2. a statistical analysis of Final Written Decisions (FWD); and 3. a "combined analysis," that looks at the multitude of outcomes possible in the IPR procedure, including settlement and request for adverse judgment.

At the ID, the PTAB can decide to institute trial on all, some, or none of the claims challenged by an IPR petition. In this analysis, we considered a denial of institution on all challenged claims to be a win for the patent owner, and any institution (on all or only some of the challenged claims) to be a loss.[4]

Similarly, at the FWD, the PTAB can decide to cancel all, some of, or none of the instituted claims. In this analysis, we considered a cancellation of less than all instituted claims to be a win for the patent owner, and a cancellation of all instituted claims to be a loss.

To quantify the results in our ID and FWD analysis, we created contingency tables and used statistical tests to see if any differences in outcome had statistical significance. The contingency tables include data for 3,963 institution decisions for petitions filed between September 16, 2012 and September 7, 2016. They involved 2,592 unique patents. The FWD contingency tables include data on 1,385 FWDs issued between November 13, 2013 and December 31, 2016. These FWDs included petitions filed as late as April 28, 2016. They involved 964 unique patents.

Joinder of petitions pursuant to 35 U.S.C. § 315(c) complicated our analysis. For the ID analysis, we accounted for joinder as follows. If two petitions were filed against one patent, we considered that to be two separate, independent events.[5] If several petitions were joined, we ignored that joinder in our ID analysis, and we counted the results of any ID once per petition. We recognize that this could lead to "double-counting" of events, but we determined that the net effect of this double counting is small. Importantly, it should not affect the comparisons between subgroups, because any error should act consistently in all our data subsets.[6]

With respect to the effect of joinder on the FWD analysis, we tested two approaches. In the first approach, we used the number of favorable FWDs assigned to a given set of patents and divided by the total number of FWDs made for that set. Illustratively, if FWD No. 1 cancelled no claims, and a FWD issued for two joined petitions cancelled all claims, then we would compute a patent owner win rate at FWD of 1/3. The second approach collapsed all joined petitions into one. So, for the example just given, the patent owner win rate would be 1/2. Broadly speaking, our results do not depend on the approach used. For our reported results, we adopted the second approach.

Our "combined analyses" combined the effects of the ID and the FWD. We did this in three ways. Our first combined analysis, "C1," used simple mathematics to combine the ID and FWD statistics. This permitted us to compute the net effect of the ID and the FWD, making the assumption that every patent for which there was an ID reached FWD. The two other combined analyses, "C2" and "C3," factored in all the possible endpoints of the IPR process, including settlement and request for adverse judgment. They made opposite assumptions about whether these other endpoints favored the patent owner or the petitioner.

The C1 combined analysis was computed as follows: A simple formula gives the probability that a patent owner

3. Links to these data sources are found in the Appendix, § 1.

4. We do, however, provide data relevant to considering a partial institution as a win for the patent owner.

5. Our future work will test this assumption.

6. We observed little or no correlation between the degree of joinder and membership in any subgroup we studied.

wins an IPR when only ID and FWD are considered. If the probability the patent owner wins at ID is p_{win}^{ID} and the probability the patent owner wins at FWD is p_{win}^{FWD}, then the overall likelihood of a patent owner winning, $P^{C1}{}_{win}$, is given by Equation 1, using the variables defined in Figure 1, below:

$$p_{win}^{C_1} = \frac{B}{B+D} + \frac{D}{B+D} \cdot \frac{H+I}{H+I+J}\text{; (1)}$$

$$p_{win}^{C_1} = p_{win}^{ID} + \left(1 - p_{win}^{ID}\right) \cdot p_{win}^{FWD}.$$

The first term of $p_{win}^{C_1}$ gives the probability of winning at the institution decision (again, we assume that only a complete denial of institution is a win for the patent owner) and the second term gives the probability of winning at FWD (*i.e.*, that at least one claim is not cancelled) reduced by a factor $\left(1 - p_{win}^{ID}\right)$ that accounts for the fact that petitions that are denied do not reach the FWD stage.

As previously stated, the C2 and C3 combined analyses included the effects of all possible IPR endpoints. Because so many endpoints are possible, there is no simple formula to rely upon. Instead, to compute the probabilities $p_{win}^{C_2}$ and $p_{win}^{C_3}$ giving the probability the patent owner wins in these analyses, we first drew "flow charts" showing the various endpoints in the IPR process. We associated the relative likelihood of a patent having a given characteristics ending up in each of the possible endpoints. Then, we summed those endpoints in the flow chart that are favorable for the patent owner.

This raises the obvious question, what is "favorable" for the patent owner? Some answers are clear. Dismissal of a matter, for example if the petitioner stopped participating, is clearly a win for the patent owner. And a request for adverse judgment is clearly a loss for the patent owner. Settlement, however is a more complex question, since settlement often reflects a compromise of the parties' positions, and could reflect either a win or a loss for the patent owner. After settlement of an IPR, the challenged claims all survive. But the patent owner may have settled with the petitioner in related litigation, and surrendered the right to sue on the challenged claims. Further, depending on the stage at which the IPR settled, the record may outline an invalidity argument upon which future infringers may rely to the detriment of the patent owner.

To account for these different views a patent owner could take of settlement, we created the C2 and C3 combined analyses. These analyses bracket the patent owner outcomes. The C2 combined analysis views settlements as losses for the patent owner. The C3 combined analysis views settlements as wins for the patent owner. See Figure 1.

For example, for the set of patents under study (IPR patents), the numerical value of "B" gives the fraction of those IPR patents for which institution was denied. This figure shows how the statistics for C2 and C3 can be computed. $p_{win}^{C_2}$ gives the probability for the patent owner winning in the C2 analysis. It is comprised of these endpoints, as shown in the figure: B (not instituted), H (all claims survive at FWD) and I (mixed at FWD). In other words,

$$p_{win}^{C_2} = \frac{B+H+I}{A}$$

The formula for $p_{win}^{C_3}$ is similar, but it adds in the probabilities of settlement: C (settlement pre-ID) and F (settlement post-ID):

$$p_{win}^{C_3} = \frac{B+C+F+H+I}{A}$$

For both these formulas, the endpoints not included, G (adverse judgment) and J (all challenged claims cancelled at FWD), are both losses for the patent owner.

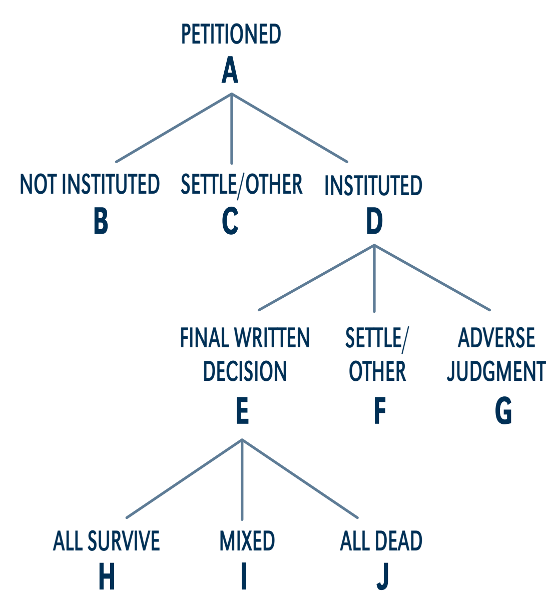
Characteristics of Patents Studied

We characterized patents involved in IPR proceedings in several ways. First, to establish a baseline, we considered how IPR patents are similar to patents in general (non-IPR patents), independent of any IPR outcome. It turns out that some characteristics that readily distinguish IPR patents from non-IPR patents do not similarly distinguish patents that fare well in IPRs from those that do not.

We analyzed the outcomes used in the ID, FWD, and

Figure 1. Flow Chart Used To Compute The Relevant Statistics

Figure 1 shows the flow chart we used to compute the relevant statistics. Each endpoint has an empirically measured probability assigned to it.



combined analyses as a function of many patent characteristics. The characteristics studied are as follows:

1. Each patent's technology;
2. The number and type of forward references for each patent;
3. Each patent's provenance, including:
   a. measures describing both the first assignee and the assignee on the date of petition in terms of size, type, and experience with patents, and
   b. changes in patent ownership from first assignee to the assignee on the petition date; and
4. The number of office actions each patent under went before issuance during prosecution of the patent and any reissues.

Technology of the IPR Patents

When we analyzed outcomes as a function of each patent's technology, we generally relied on the USPTO's classification of a patent into a Technology Center (TC). This classification is described in the Appendix §2. We have conducted similar analyses reflecting a more detailed assessment of outcomes at the technology center unit (TCU), and general art unit (GAU) level. That more detailed assessment is, however, beyond the scope of the present paper.

For patents classified by the USPTO into TC 3600, "Transportation, Construction, Electronic Commerce, Agriculture, National Security and License & Review," we use a modified classification and rely upon the more granular classification— one based on the USPTO's technology center unit (TCU) level. We classified patents belonging to Technology Center Units (TCUs) 3620 ("Electronic Commerce"), 3680 ("Business Methods"), and 3690 ("Business Methods—Finance") as belonging to a "3600 E-commerce" set. We classified patents belonging to the remaining TCUs of TC 3600 as belonging to a "3600-Traditional" set. More details are found in the Appendix § 2.

Table 1 shows this technology classification and our nomenclature for each technology. The USPTO's names for each TC are given in the Appendix § 2.

Table 1. Flow Chart Template

Technology Center	As referred to in this paper
1600	Biotech/Organic Chem
1700	Chemical/Materials Engineering
2100	Computer Architecture
2400	Networking
2600	Communications
2800	Semiconductors
2900	Design patents—*Not analyzed*
3600	E-Commerce (3600EC) and Traditional (3600Trad)
3700	Mechanical engineering/Medical devices

Provenance

Generally, we use the word "provenance" to reference characteristics of a patent's owner, whether at the time of the patent filing or at the time of an IPR petition. We identified the provenance of a patent in several ways, including the following:

- Determining the "Patent Ownership" of the patent's first assignee;
- Determining whether the patent was filed by an educational or research institution;
- Determining whether the patent was filed by a small entity; and
- Determining whether at the time the IPR petition was filed the patent was assigned to an entity different than the first assignee.

We created a Patent Ownership measurement to estimate the level of experience with the patenting process possessed by each IPR patent's assignee. We hypothesized that the more patents issued to an assignee, the stronger the assignee's patents would be. To determine the Patent Ownership for each first assignee (the assignee first identified in the file history, as recorded by the IPDD database), we ranked the assignees of each year's issued patents into quartile groups, where the first quartile was issued the largest quartile of patents in that year, the second quartile was issued the next quartile of patents, and so on. For example, in 2010, the first quartile consisted of 31 firms, including Samsung and IBM, the second quartile consisted of 231 firms, the third quartile had 3,742 firms, and the fourth quartile had 43,099 firms.

We determined whether a patent was filed by a small entity using a field in the IPDD database that corresponds to the "discounted" or "small entity status" discount for USPTO fees specified by 37 CFR 1.27. We identified educational and research institutions by first building a list of assignees having "university," "college," "school," "foundation" or "institute" in the name and then manually verifying each entry. When an assignee was identified as both a small entity and an educational institution, we reclassified it as an educational and research institution, but not as a small entity. We determined the current assignee (CA) of an IPR patent using the IPDD database, defining "current" to be the date the IPR petition was filed.

Forward References

We hypothesized that patents having many forward references would be stronger than average because they may contain the earliest disclosures of inventive limitations. We extracted the number of forward reference citations for each subject patent from the IPDD database. For IPR patents, we determined the number of forward references to the subject patent made by U.S. patents on or before the date of the subject patent's IPR petition, as further described in the Appendix § 5. (When multiple petitions were joined pursuant to 35 U.S.C. § 315 (c), we used the date of the earliest petition as the cutoff date.) When a subject patent and its associated publication were both cited by a given patent, we counted that as only one forward reference.

Beyond simply counting an IPR patent's forward references, we also computed two additional measures for forward references. First, we computed the number of forward references to an IPR patent by eliminating what we called "self-citations." Self-citations are forward references to a patent made by that patent's first assignee.[7] Second, we counted "Examiner Citations" by using the IPDD database to identify citations made to a patent by a patent examiner during the prosecution of any other application before the date of the relevant IPR petition.

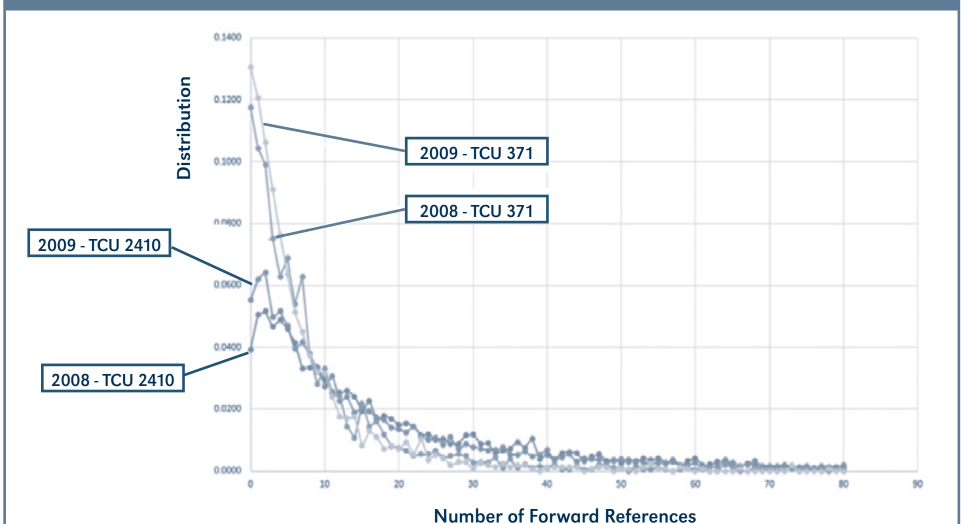
Raw numbers of forward references depend on factors including the patent's age and the level of inventive activity in the technology of the subject patent. It is likely that a typical five-year-old patent in an active TCU will have more forward references than a typical two-year-old patent in a TCU that is less active. A patent's age impacts the number of forward references expected for a given patent: new patents do not have much time to accumulate references and old patents have a lot of time to do so. Accordingly, we controlled for patent age when analyzing forward references.

We similarly examined the forward reference distribution of various TCUs for patents issued in the same year and found a significant variation between TCUs. Illustratively, Figure 2 compares forward reference distribution for TCUs 2410 and 37I respectively, for patents issued in 2008 and 2009.[8] Justifying our decision to control forward references, all four distributions are different. First, as expected, the older patents tend to have more forward references than do the newer patents. Further, and notably, the two distributions have different shapes—TCU 37I patents (which covers refrigeration) have significantly fewer forward references than do the TCU 2410 patents (which cover VoIP communications). See Figure 2.

We controlled our forward reference measures (total forward references, forward references without self-citations, and examiner references) in the following manner: For each IPR patent, we identified the patent's "peer group." We defined a peer of a patent as one that both (1) was issued in the same year as the subject patent, and (2) was a member of the patent class(es) the patent's examiner searched during the patent's prosecution.

We next determined the distribution of forward references to each of these peer patents, using the same date criteria as we used for the subject patent. We took this distribution to be the model for the number of expected forward references. This model had a mean and a standard deviation. We then computed the z-score or the log z-score for the subject

Figure 2. Distribution Of Forward References For TCUs 2410 And 371, Over 2008 And 2009



7. We identified these assignees using the "normalized name" field of the IPDD database. We also built our own list of variants of assigned names to help identify self-citations.

8. TCU 2410 is entitled "Multiplex & VoIP" and 37I is entitled "Refrigeration, Vaporization, Ventilation, and Combustion."

patent using these formulas:

$$Z(FR_i) = \frac{FR_i - \mu_{pgi}^{FR}}{\sigma_{pgi}^{FR}}$$

$$Z(\ln(FR_i+1)) = \frac{\ln(FR_i+1) - \mu_{pgi}^{\ln FR}}{\sigma_{pgi}^{\ln FR}}$$

(We refer to 2(In(FR))as "log Z-score")

These formulas express how much the subject patent's forward reference count is above or below the model distribution's mean, measured in terms of the model distribution's standard deviation.

Z-score and log z-scores are commonly used to compare data sets where (like ours) the statistics of a distribution depend on confounding variables.[9] In this case, as noted above, the variables are the patent age (measured in years post-issue) and the patented technology (measured by the previously-defined TCU-based peer groups).

Office Actions

For each IPR patent, we determined the number of office actions that occurred during patent prosecution by counting the entries in the IPDD database. We provide additional details in the Appendix § 3. As with forward references, for each IPR patent, we analyzed office actions with z-scores and log z-scores based on a comparison to each IPR patent's peer group.

Statistical methods

For analyses based on contingency tables, we used the t-test with a p value of 0.05. When indicated, we corrected for multiple comparisons using the Benjamini-Hochberg procedure with a false discovery rate of 0.1. Appendix § 4 contains more details on our statistical methodology.

Results

How do IPR patents compare to patents as a whole?

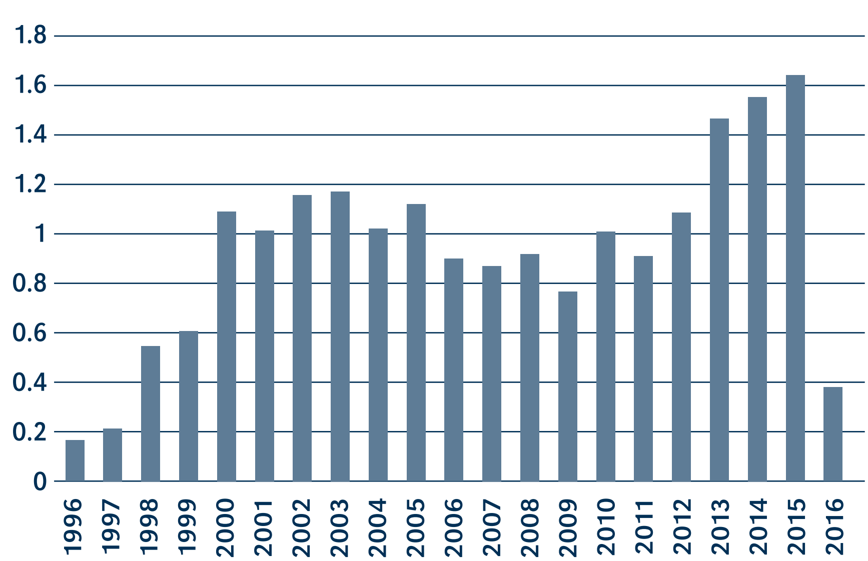
IPR patents are different in a statistically significant way from non-IPR patents.

Age of patents at the time an IPR petition was filed

The age of a patent has a strong effect on whether it will be subject to an IPR. The count of IPR patents by age correlates fairly well with the number of patents issued for each year (correcting for the number of patents cancelled because of failure to pay maintenance fees). Figure 3 shows, by issue year, the number of IPRs filed in 2016 expressed as a fraction of the number of patents issued each year, for calendar years 1996 to 2016.[10] We scaled the ratio so that its average value over the years 2000-2012 is 1.0.

Given that scaling, if IPR patents were randomly selected from issued patents, the curve in Figure 3 would be flat, and have a constant value of 1.0. In fact, however, the curve varies from that value in three time periods. For 2016, the value is far less than 1.0. This represents the facts that (1) on average, throughout 2016, only half the patents ultimately issued were in fact issued by the petition's filing date, and (2) litigation of patents (and any ensuing IPR) takes some time to get started—accordingly IPR petitions will lag the patent's issuance year. The oldest patents (1996-1999) are, not surprisingly, underrepresented in IPR petitions as they have little life remaining.

Figure 3. Patent Age At IPR Filing And Issued-By-Year Ratio



9. Appendix § 4 contains discussion of the rationale for using z-scores.

10. We also deleted from the patent count patents that expired for lack of maintenance fees payment and patents otherwise cancelled. We used the IPDD database to make these deletions.

Tech Center	TC petitoned	TC pct of pet
2600	988	17.8%
2800	900	16.2%
2400	897	16.1%
3700	694	12.5%
1600	542	9.7%
2100	509	9.2%
3600Trad	499	9.0%
1700	395	7.1%
3600EC	137	2.5%
Total	5561	

Table 2. IPR Patent Population Of TCs

Most notably, patents issued in the three calendar years before the year of IPR filing are overrepresented in IPR petitions. As shown in Figure 3, this overrepresentation is approximately 50 percent above the previous 13 years.

The Patented Technology

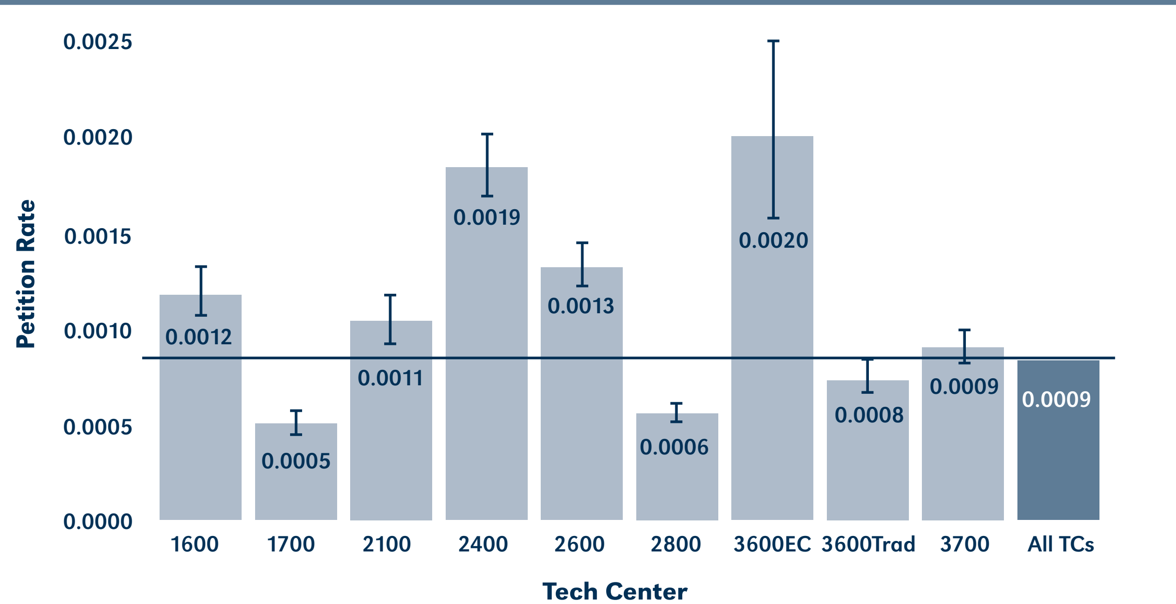
IPR patents belonging to certain TCs predominate in IPR proceedings. Table 2 shows the counts for each TC for the 5,561 petitions filed through December 31, 2016. TCs 2600 (Communications), 2800 (Semiconductors), and 2400 (Networking) are the most commonly petitioned, together making up approximately 50 percent of petitions.

Some patent technologies are represented among IPRs in proportions different from their proportions in issued patents. Figure 4 illustrates how the proportion of IPR petitions differs from the proportion of patents issued in each TC. The horizontal line shows the average rate: 0.09 percent of all patents are involved in IPRs. Relative to this average value, however, some patent technologies stand out. TCs 3600 (E-commerce), 2400 (Networking), and 1600 (Biotech/Organic Chem) are especially overrepresented among IPR patents, and TCs 1700 (Chemical/Materials Engineering) and 2800 (Semiconductors) are particularly underrepresented. All differences are statistically significant. Appendix § 4 describes the statistical techniques we used to determine significance.

We computed the relative shares of patents assigned to educational and research institutions, small entities, and other entities by TC. Table 3 presents the results of this calculation. It shows the ratio of IPR petitions filed in each TC, for IPR patents owned by each of the three classes, as a ratio calculated to the expected value if patents were randomly distributed across TC.

In Table 3 a value greater than one indicates that the TC is overrepresented in the indicated entity status. For example, the table shows that educational insti-

Figure 4. Over/Under Representation Of TCs In IPR Patents Relative To Issued Patents



tutions have a disproportionately large share of IPR patents belonging to TC 1600 (Biotech/Organic Chem; ratio = 3.1) and a disproportionately small share of IPR patents belonging to TC 2100 (Computer Architecture; ratio = 0.2).

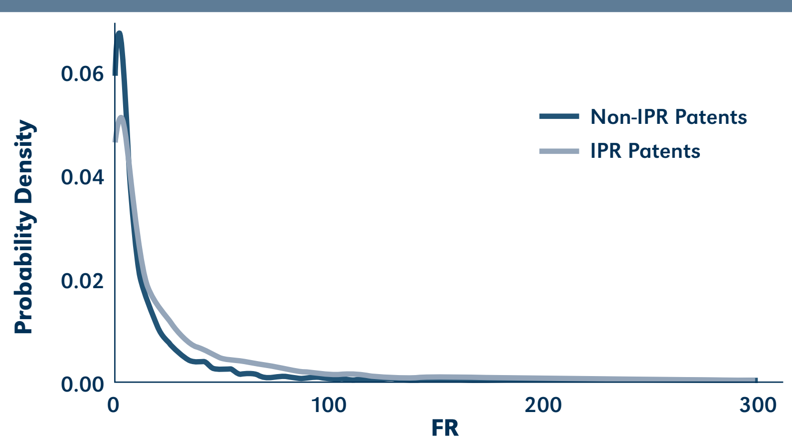
Table 3. Over/Under-Representation In TCs By Entity Status

TC	University	Small-entity	Other
1600	3.1	0.4	1.2
1700	1.5	1.3	0.8
2100	0.2	1.1	0.9
2400	0.5	0.9	1.1
2600	0.8	1.1	1.0
2800	1.4	1.1	1.0
3600EC	0.0	1.2	0.9
3600Trad	0.0	0.9	1.1
3700	1.1	1.1	1.0

Table 4. IPR Patent Ownership By Quartile

Patent Ownership Quartile	Share of IPR Patents
1 (most patents)	5%
2	11%
3	34%
4 (least patents)	51%
Total	100%

Figure 5. Forward Reference Distribution Between IPR And Non-IPR Patents



Provenance

The distribution of IPR patents is strikingly out of proportion to the Patent Ownership measure we created. If IPR patents were randomly distributed among first assignees in proportion to the number of patents each was awarded, then one-quarter of the IPR patents would be associated with each Patent Ownership quartile. But as Table 4 shows, the distribution is not uniform: assignees in the top quartile of patent ownership own less than five percent of the IPR patents, and firms with the lowest ownership of patents have more than 51 percent of IPR patents.

Forward References

IPR patents are also statistically different from non-IPR patents with respect to forward references. Figure 5 shows the difference between forward reference counts for IPR patents and non-IPR patents, averaging over all TCUs and issue dates.[11] In this figure, the non-IPR patent distribution is peaked to the left, and the IPR patent has a longer tail: IPR patents have more forward references than do non-IPR patents. This conclusion is supported by our analysis that corrected for patent-issue year and TCU, using both z-scores and log z-scores. The Appendix § 5 summarizes our analyses using these methodologies.[12]

Office Actions

Further, IPR patents are statistically different from non-IPR patents with respect to the number of office actions made before issuance. Figure 6 shows the difference between office action counts for IPR patents and the patents in their peer group patents, averaging over all TCUs and issue dates. IPR patents have significantly more office actions than do patents as a whole. This conclusion is supported by our analysis that corrected for patent-issue year and TCU, using both z-scores and log z-scores. Appendix § 6 summarizes our analyses using these methodologies.

Differences in Winning in IPRs Based on Patent and Assignee Characteristics

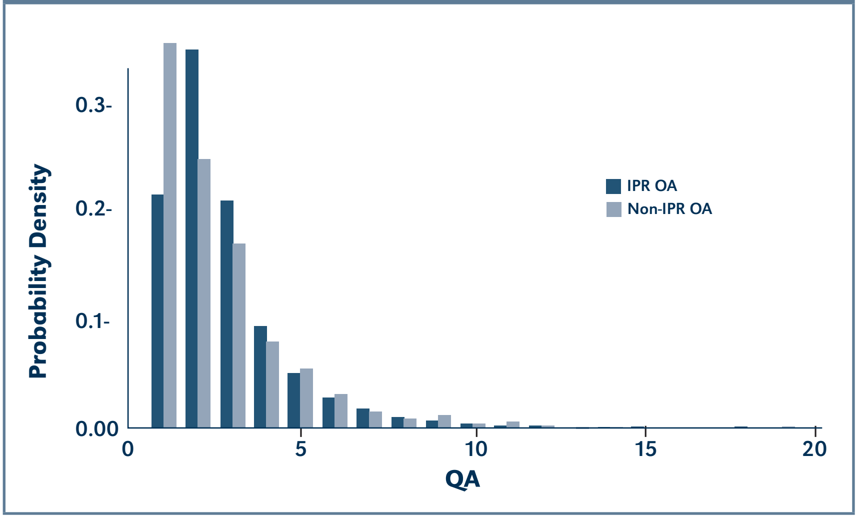
The Baseline Outcomes

Two baseline outcomes for all studied IPR petitions, are as follows:

11. To create this figure we generated one distribution for all IPR patent forward references, and then generated a second distribution for a set of peer patents for each, one peer for each IPR patent.

12. The data are plotted smoothed, so it appears that non-integral numbers of references are possible.

Figure 6. Office Actions Distribution Between IPR And Non-IPR Patents



a. 29.1 percent of IDs are decided without instituting a trial, which, using our definition, is a win for the patent owner.[13]

b. 30.3 percent of FWDs results in at least one instituted claim surviving, which under our definition, is a win for the patent owner.

Turning to the combined analysis, there are three results. Combining ID and FWD statistics using Equation 1 (and in so doing, ignoring all possible endpoints but institution and final written decision) gives the probability of winning by the end of the IPR proceeding as

$$p_{win}^{C_1} = 48.8\%$$

In other words, making the assumption that 29.1 percent of IDs fail to yield an institution (a win for the patent owner) and that the patent owner wins 30.3 percent of FWDs, assuming that nothing else happens in the IPR, then patent owners win about 50 percent of the time.[14]

Dropping the assumption that "nothing else happens," as we do in the C2 and C3 combined analyses, summing the data found in Appendix § 7 gives

$$p_{win}^{C_2} = 35.9\%$$

$$p_{win}^{C_3} = 66.2\%$$

As previously discussed, these values bracket the range of patent owner outcomes. The first value measures a patent owner win rate assuming that settlements are losses, and the second value assumes that settlements are patent owner wins.

Several observations about these results are in order. First, the probability of the patent owner winning the C1 combined analysis—which ignores settlement— $p_{win}^{C_1} = 48.8\%$, is bracketed by C2 result (35.9 percent), which counts settlements as losses, and the C3 result (66.2 percent), which counts settlements as wins. Second, these results present a picture of the IPR process that is perhaps not as dire for patent owners as commonly believed. A patent has approximately the same chance of having a claim that survives an IPR as having all challenged claims cancelled. The PTAB's boards may be death panels, but they are not terrifically efficient ones.

What Are The Signs Of A Strong Patent? Comparing Winning And Losing IPR Patents

Understanding that parties involved in patent licensing and enforcement cannot change the PTAB's approach to patents, a relevant question is "how can parties involved in patent licensing and enforcement assess which patents are most likely to survive the IPR process?" The value of this paper lies in the identification of patent characteristics that alter the chances of winning from the baseline values.

Tech Center Results

The ID Stage

As noted above, across all TCs, at the ID stage patent owners win 29.1 percent of the time. But the percent of patent owner winning varies significantly from this baseline value depending on the patents' technology. Figure 7 shows this effect. The horizontal line gives the baseline win rate, independent of TC. The TCs shaded darker have a statistically significant different rate from the average. (We use this shading scheme throughout.)

The data underlying Figure 7 are found in the Ap-

13. We did not study what overall fraction of IDs entailed institution for all claims, so we cannot give a baseline number for winning at ID if the definition of the patent owner's success includes a partial institution. However, of the FWDs, 24.6% had partial institutions. If this percentage is valid across all institutions (not just for those that reached FWD), then the baseline number for winning at ID would be 54.5% = (29.9% + 24.6%).

14. The data set used for the ID and the FWD analyses differs slightly from the data set used for the combined analyses. Accordingly, applying Equation 1 to the ID and FWD results reported on this page gives a slightly different value for $p_{win}^{C_1}$ than is reported above.

Figure 7. Win Rate At *ID vs. TC*

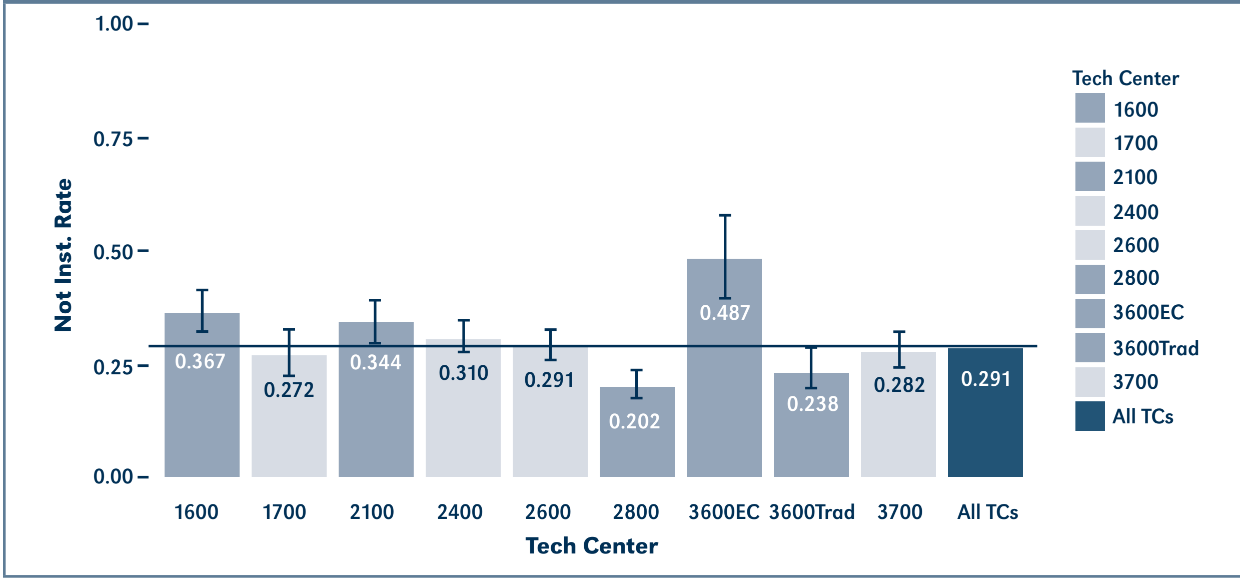
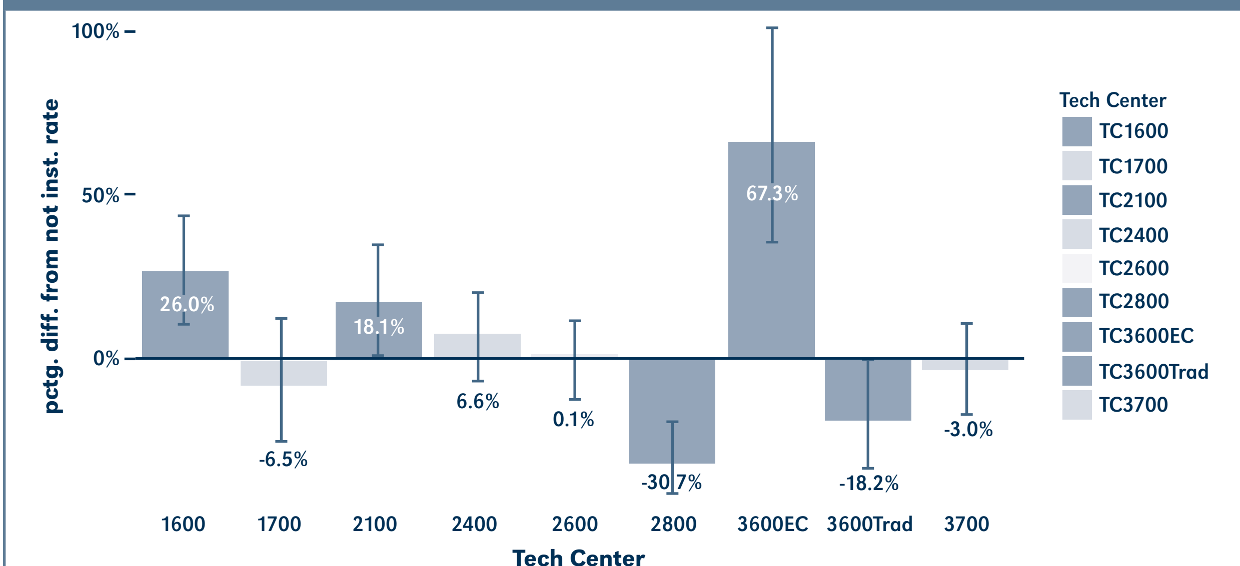


Figure 8. Win Rate At *ID vs. TC*—Relative To The Average Win Rate



pendix § 8. Figure 8 shows the same data, but computing the winning rate as a percentage above (or below) the baseline rate, 29.1 percent. The results indicate that patents in TC 3600 Electronic Commerce, TC 1600 (Biotech/Organic Chem), and TC 2100 (Computer Architecture) are most likely to win at ID. On the other hand, patents in TC 2800 (Computer Architecture) and perhaps TC 3600 Traditional are least likely to win at ID.

The FWD stage

Patent owners also win at greater than the baseline rate at the FWD stage with certain TCs. As shown in Figure 9, patent owners win at the FWD in elevated rates with TC 1600 (Biotech/Organic Chem). Figure 9, analogously to Figure 8, compares the FWD survival rate as a percentage above (or below) the average win rate, 33 percent. Compared to the baseline, the win rate for TC 1600 is elevated by 61 percent.

Figure 9. Win Rate At *FWD vs. TC*—Relative To The Average Win Rate

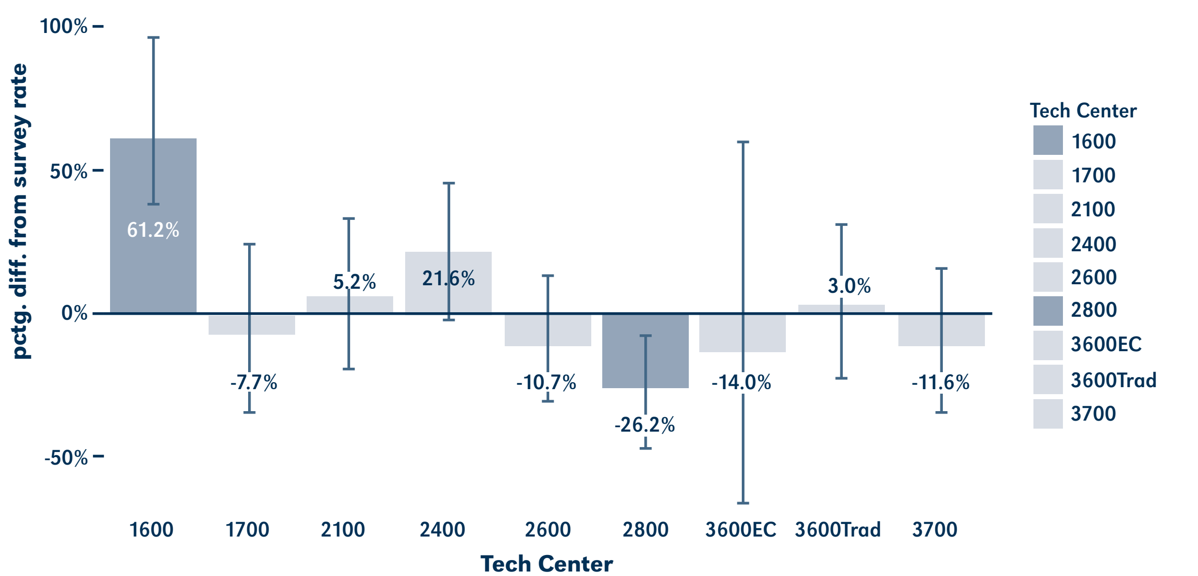


Table 5. C1-C3 Win Rates (Combined Analysis)

TC	C1	C2	C3
1600	68.5%	50.2%	76.9%
1700	34.3%	28.5%	51.0%
2100	54.6%	45.4%	66.2%
2400	58.7%	42.1%	72.4%
2600	47.4%	32.6%	69.6%
2800	36.2%	25.7%	60.0%
3600	67.7%	58.1%	76.3%
3600	44.6%	32.4%	59.4%
3700	44.0%	31.4%	66.8%
All TCs	48.8%	35.9%	66.2%

Key: Above average win rate | Below average win rate

On the other hand, patent owners do worse with TC 2800 (Semiconductors). For this TC, the patent owner win rate is only 74 percent of the baseline value. Even within TCs that do not as a whole show a statistically significant difference in win rates, we have observed significant differences for specific technologies at the TCU or GAU level. That analysis, however, is beyond the scope of the present paper.

The Combined Analysis

Turning to the combined analysis, Table 5 shows the results for C1, C2 and C3 by TC. Light blue numbers indicate a patent owner win rate above average for the TC, and dark blue indicates a lower-than-average win rate.

As shown in Table 5, almost all TCs are associated with winning at the end of the IPR process with either statistically significant elevated rates or with statistically significant depressed rates, compared to the baseline. Figure 10 illustrates that point. It shows the patent owner win rate as a percentage above (or below) the average rate for the data set using the C1 analysis. Here, TC 1600 (Biotech/Organic Chem), 3600 E-commerce, TC 2400 (Networking), and TC 2100 (Computer Architecture) are associated with statistically significant elevated win rates. TC 1700 (Chemical/Materials Engineering), 2800 (Semiconductors), and 3700 (Mechanical engineering/Medical devices) are similarly associated with statistically significant depressed win rates.

Provenance Results

When classifying IPR patents by the status—educational and research institution, other small entity, or other—we found a small and not-always-consistent effect that small entity patent owners win less than average. Figures 11-16 illustrate this point. Figures 11-13 give the results for ID, FWD, and C2 respectively, for IPR patents classified according to first assignee. Figures 14-16 give the respective results classified according to current assignee. These figures show that generally small entities fared worse than average with respect to ID, FWD, and the combined analysis.

A possible explanation for small entities' lower win rate is based on the relative under-participation of

Figure 10. Win Rate At *C2 vs. TC*—Relative To The Average Win Rate

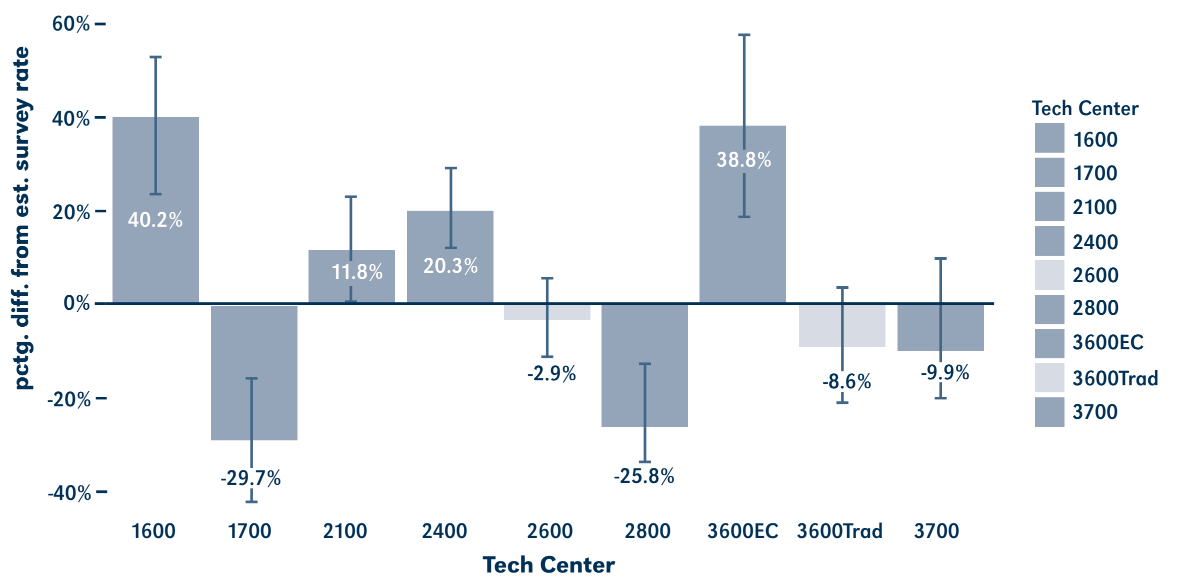


Figure 11. ID Analysis For First Assignee...C2 Analysis For Current Assignee

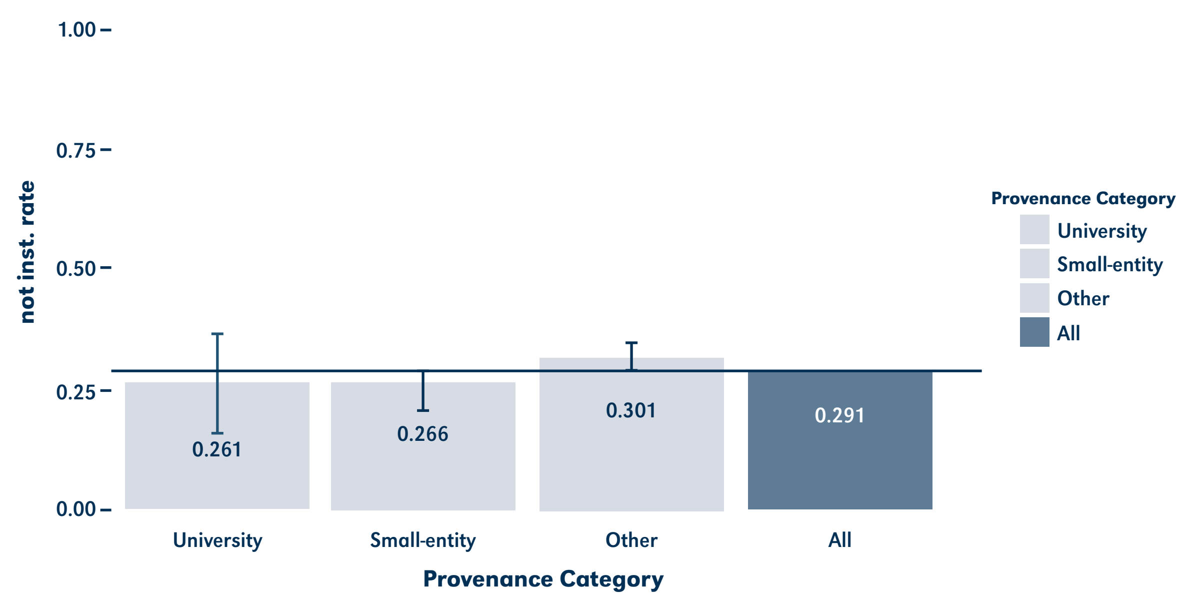


Figure 12. FWD Analysis For First Assignee...C2 Analysis For Current Assignee

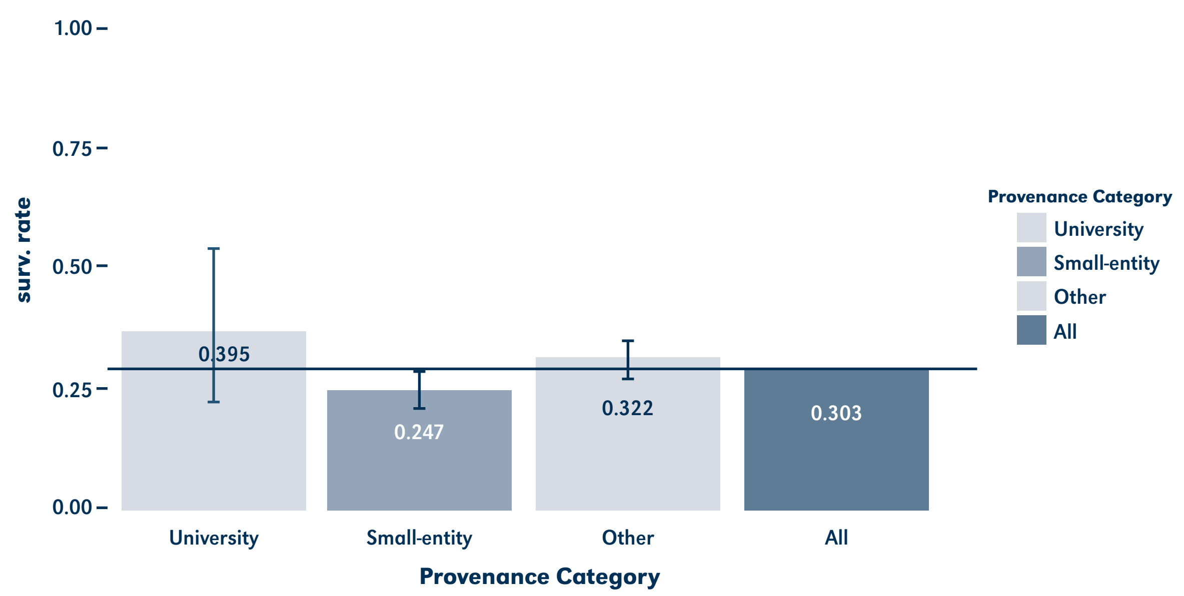


Figure 13. C2 Analysis For First Assignee...C2 Analysis For Current Assignee

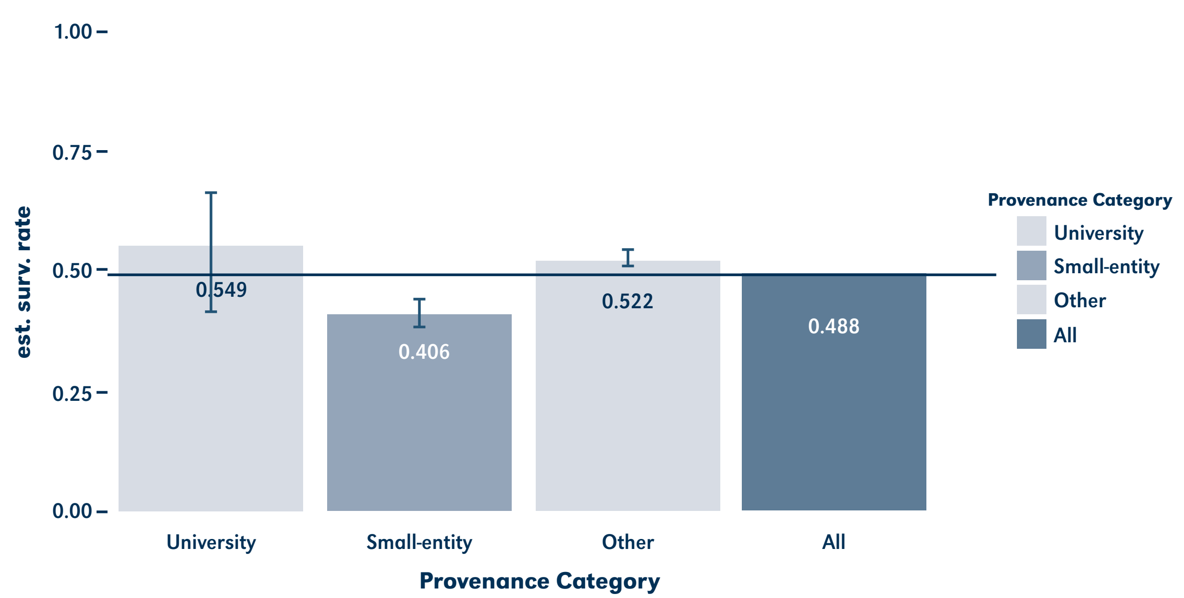


Figure 14. ID Analysis For First Assignee...C2 Analysis For Current Assignee

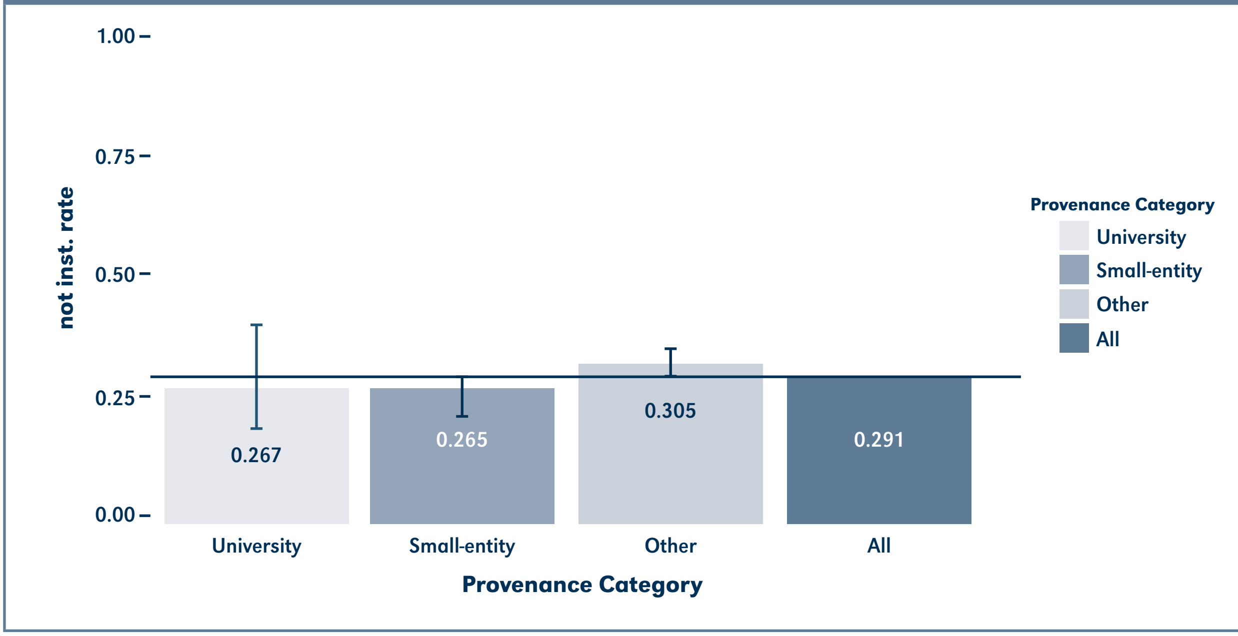


Figure 15. ID Analysis For First Assignee...C2 Analysis For Current Assignee

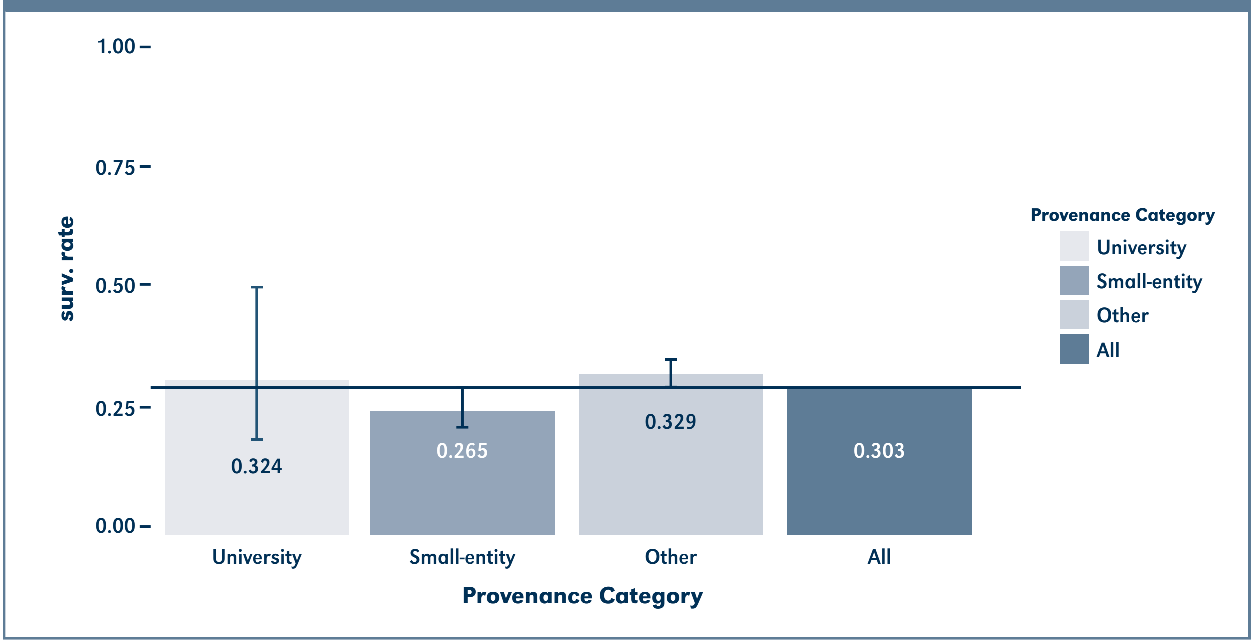


Figure 16. C2 Analysis For First Assignee...C2 Analysis For Current Assignee

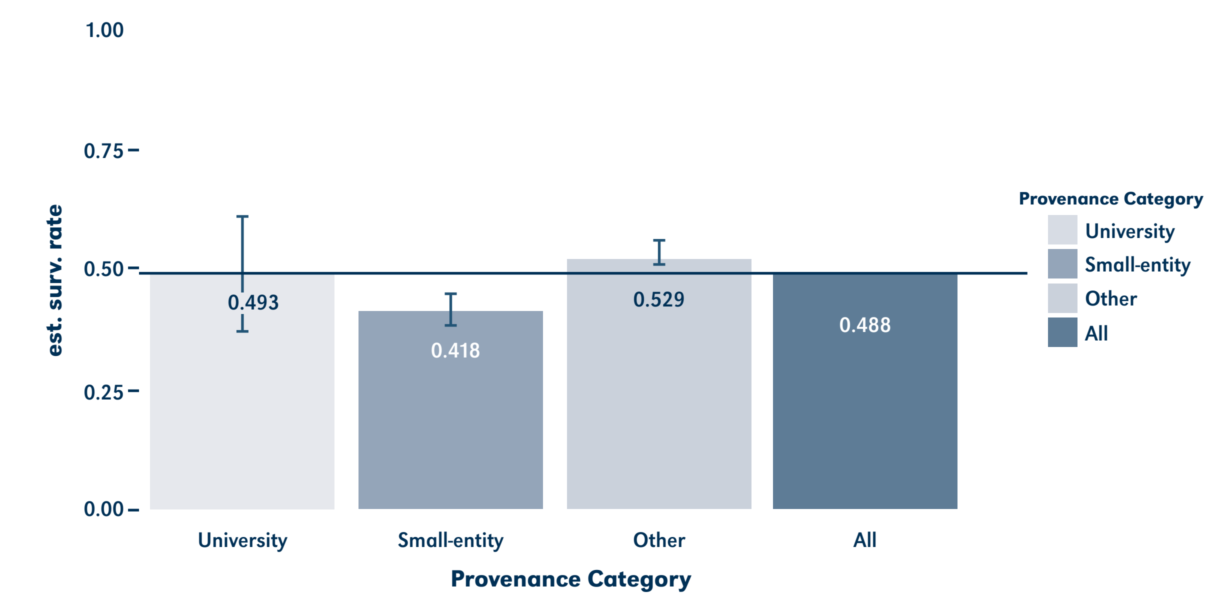


Figure 17. Win vs. Patent Ownership Quartile

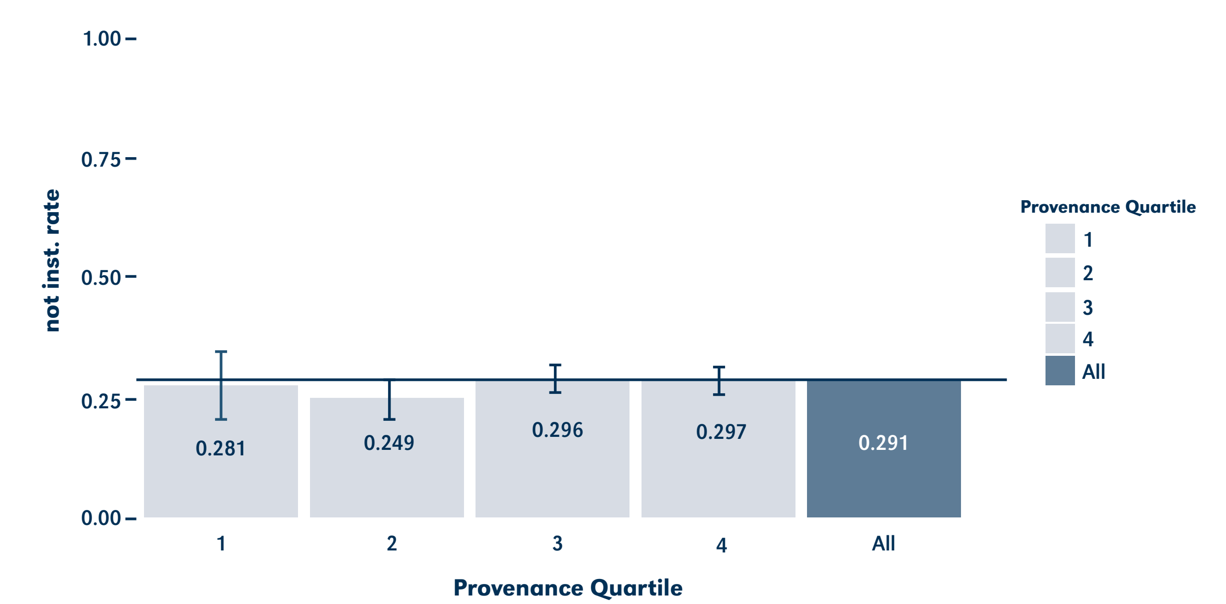
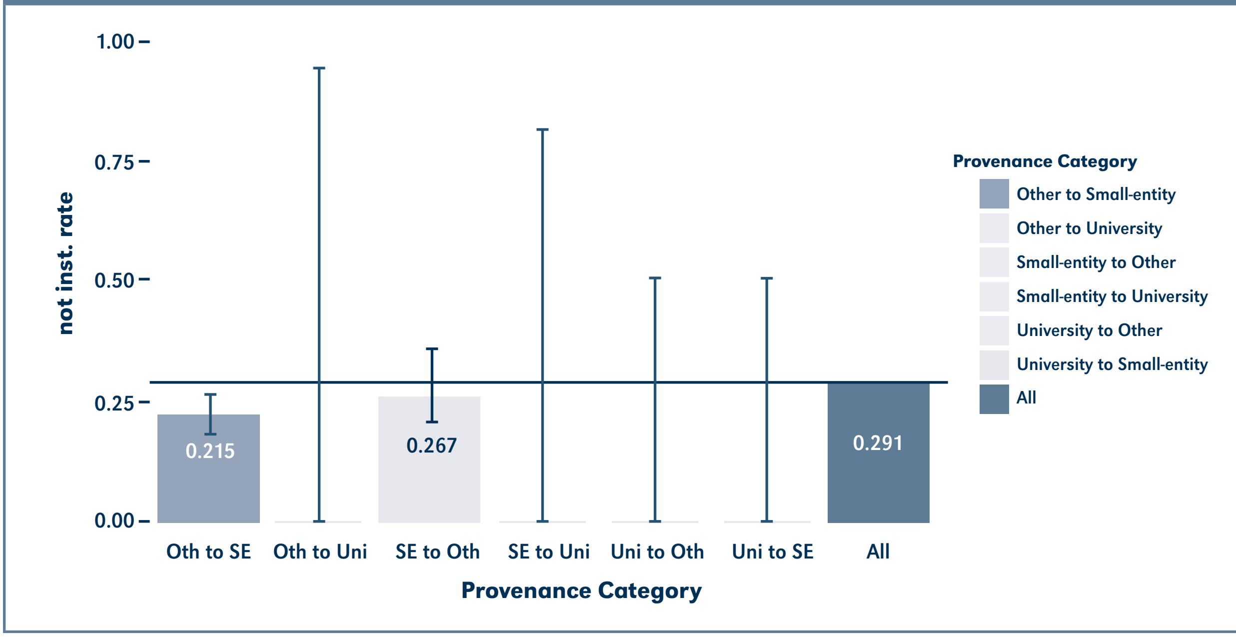


Figure 18. Sample Graph Showing Change In Entity-Status On Win Rate



small entities with patents in the TCs that have high patent owner success rates. As shown in Table 3, small entities have only 40 percent of the TC 1600 (Biotech/Organic Chem) patents as would be expected if distribution over TC were random. Further, according to the results presented in Figure 7, patent owner win rates in that TC are appreciably above the average win rate. Accordingly, small entities' win rate suffers because, when compared to average patent owners, they seem to own fewer of the stronger patents.

We found that our Patent Ownership measure, based on quartiles of patent ownership, had no predictive power. Figure 17 shows one illustration of this point. It shows that success at ID did not depend on Patent Experience in a statistically significant way. We found similar results for the FWD and the combined analyses. These results are found in the Appendix § 9.

IPR Patent Transfers and Provenance

We studied approximately 350 petitions that involved IPR patents where the current assignee had a different status than the patent's first assignee.[15] For example, this would include patents where the first assignee was neither an educational/research institution nor a small entity (we called this the "other" status) and the current assignee was a small entity. The two largest changes in provenance were 1. "other" to "small entity" totalling 247 petitions, and 2. "small entity" to "other" totalling 90 petitions.

In this study, we did not find large or consistent effects. As Figure 18 shows, small entity win rates at the ID stage were statistically lower than average. The result is consistent with the observation above that small entities are slightly less likely than average to win at ID.

On the other hand, these "other" to "small entity" transfers were associated with a higher rate of surviving final written decision than average. (See Appendix § 10.) They also, consistent with our previous observation about small entity patents, had a lower win rate in the C2 combined analysis than average.

Forward Reference Results

When comparing the forward references for instituted petitions to those that were not instituted, we found no significant difference between the two sets.

Table 6. Forward Reference Results

	μ_1	σ_1	n_1	μ_2	σ_2	n_2	p-value
A	2.61	1.62	2810	2.60	1.67	1153	0.852
B1	2.75	1.66	409	2.70	1.57	976	0.599
B2	2.67	1.62	367	2.67	1.60	843	0.964
C	2.69	1.64	1115	2.70	1.56	1051	0.835

15. We also eliminated petitions involving patents that appeared based on the entities' names to have been transferred between two related owners—*e.g.*, a patent transfer between Michael Smith and Michael Smith LLC.

The same conclusion that forward references have no significant effect is true for petitions surviving FWD and for those surviving IPR in the combined analyses. This observation is notable because, as shown above, IPR patents differ from non-IPR patents significantly, in terms of forward references. Thus, it appears that forward reference counts significantly impact which patents are selected for licensing and enforcement—and thereby end up in the IPR process—but, once challenged, have little or no impact on the success of the patent during the IPR process.

Table 6 shows a statistical analysis of this outcome. Any differences in distributions are insignificant ($p>0.5$).

Figure 19 confirms this conclusion. It shows how close the log z-scores for winning and losing patents in the ID stage are. Graphs using other measures similarly failed to show statistically significant differences. These results, including our analysis of self-citations and examiner citations, are found in the Appendix § 11.

Figure 19. Forward Reference Results

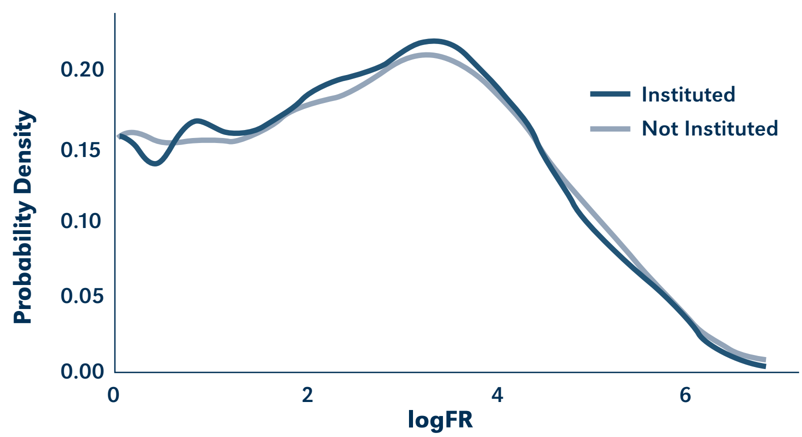


Table 7. Office Action Results

	μ_1	σ_1	n_1	μ_2	σ_2	n_2	p-value	BH Signif.
Dataset A	0.290	2.05	2810	0.268	1.82	1153	0.743	
Dataset B1	0.308	2.28	409	0.418	2.52	976	0.429	
Dataset B2	0.337	2.38	367	0.439	2.66	843	0.510	
Dataset C	0.363	2.22	1115	0.381	2.45	1051	0.860	

Office Action Results

As with our analysis of forward references where we found that IPR patents differ from non-IPR patents, but that forward references do not affect winning at the various IPR endpoints, so is it with office actions. Table 7 shows that the number of office actions has no effect on winning under any model tested. For all comparisons, $p>0.4$. Additional results are found in the Appendix § 12.

Conclusions

Assessing risks associated with patent licensing and enforcement is a highly complex, fact-specific undertaking. Statistical analysis alone will never supplant the need for detailed qualitative assessment by experienced counsel and technical professionals. As illustrated above, however, a detailed quantitative assessment of patent characteristics can help forecast success or failure at the USPTO's PTAB. Assessments like this can provide valuable data-driven inputs to patent owners, capital investors, and potential licensees tasked with evaluating risk in a rapidly transforming patent environment.

Acknowledgements

The authors thank their colleagues at Robins Kaplan LLP, without whose assistance the work would not have been possible. In particular, the authors would like to thank Ian LaForge, Data Scientist, and Jill Parrott, Patent Trial Group Clerk.

Disclaimer

The opinions expressed herein are those of the authors alone, and do not constitute the position of Robins Kaplan LLP. Neither are they legal advice. ■

Available at Social Science Research Network (SSRN): *https://ssrn.com/abstract=3103202*

Appendix

List of Figures

Figure	Title	Page
1	Flow Chart template	31
2	Representative forward references distributions	33
3	Patent age at IPR filing and issued-by-year ratio	34
4	Over/under representation of TCs in IPR patents relative to issued patents	35
5	Forward reference distribution between IPR and non-IPR patents	36
6	Office actions distribution between IPR and non-IPR patents	37
7	Win rate at ID vs. TC	38
8	Win rate at ID vs. TC – relative to the average win rate	38
9	Win rate at FWD vs. TC – relative to the average win rate	39
10	Win rate at C2 vs. TC – relative to the average win rate	40
11-16	ID analysis for First Assignee . . . C2 analysis for Current Assignee	40-43
17	Win vs. patent ownership quartile	43
18	Sample graph showing change in entity-status on win rate	44
19	Forward reference results	45
A1	Density plot of FR and peer FR	51
A2	Density plot of Log FR and peer Log FR	51
A3	Actual Survival Rates of IPR by TC	52
A4	Actual Survival Rates of IPR by TC (Settlement as Favorable)	53
A5	Summary of Institution Decisions by TC	54
A6	Summary of Institution Decisions by CA Provenance Category	54
A7	Summary of Final Written Decisions by CA Provenance Category (All Petitions)	55
A8	Summary of Actual Survival of IPR by CA Provenance Category	56
A9	Summary of Institution Decisions by CA Provenance Quartile	56
A10	Summary of Final Written Decisions by CA Provenance Quartile (All Petitions)	57
A11	Summary of Actual Survival of IPR by CA Provenance Quartile	58
A12	Summary of Institution Decisions by Provenance Entity Status Change	58
A13	Summary of Final Written Decisions by Provenance Entity Status Change (All Petitions)	59
A14	Summary of Actual Survival of IPR by Provenance Entity Status Change	60
A15	Density Plot of Log FR and Log Peer FR	61
A16	Histogram of Office Actions by Outcome Variable	62

List of Tables

Table	Title	Page
1	TC names	32
2	IPR Patent population of TCs	35
3	Over/under-representation in TCs by entity status	36
4	IPR patent ownership by quartile	36
5	C1-C3 win rates (combined analysis)	39
6	Forward reference results	44
7	Office action results	45
A1	TC names used by the USPTO	48
A2	Summary Paired T-Tests of Office Actions	52
A3	Binomial Tests on Actual Survival of IPR by TC (FDR = 0.1)	52
A4	Binomial Tests on Actual Survival of IPR by TC (Settlement as Favorable) (FDR = 0.1)	53
A5	Binomial Tests of Institution Decisions by TC (FDR = 0.1)	53
A6	Binomial Tests of Institution Decisions by CA Provenance Category (FDR = 0.1)	54
A7	Binomial Tests of Final Written Decisions by CA Provenance Category (All Petitions) (FDR = 0.1)	55
A8	Binomial Tests of Actual Survival of IPR by CA Provenance Category (FDR = 0.1)	55
A9	Binomial Tests of Institution Decisions by CA Provenance Quartile (FDR = 0.1)	56
A10	Binomial Tests of Final Written Decisions by CA Provenance Quartile (All Petitions) (FDR = 0.1)	57
A11	Binomial Tests of Actual Survival of IPR by CA Provenance Quartile (FDR = 0.1)	57
A12	Binomial Tests of Institution Decisions by Provenance Entity Status Change (FDR = 0.1)	57
A13	Binomial Tests of Final Written Decisions by Provenance Entity Status Change (All Petitions) (FDR = 0.1)	58
A14	Binomial Tests of Actual Survival of IPR by Provenance Entity Status Change (FDR = 0.1)	59
A15	Summary of Paired T-Tests of Forward References	60
A16	Summary of T-Tests of Log of Office Actions by Outcome Variable	61
A17	Summary of T-Tests of Z of Log of Office Actions by Outcome Variable	61
A1B	Binomial Tests of Institution Decisions by Number of Office Actions (FDR = 0.1)	62
A2B	Binomial Tests of Final Written Decisions by Number of Office Actions (All Petitions) (FDR = 0.1)	62
A3B	Binomial Tests of Survival of IPR by Number of Office Actions (FDR = 0.1)	62

§ 1 Data Sources

Links to the data sources we used follow:

PTAB's End-to-End Database
https://ptab.uspto.gov/#/login

PTAB's API
https: //ptabdataui.uspto.gov/#/introduction

LexisNexis IP DataDirect Database
https://internationalsales.lexisnexis.com/products/ip-data-direct

§ 2 Technology Centers—How We Classified IPR Patents' Technology

The following table gives the names for each of the Technology Centers used by the USPTO:[1] See Table A1.

We used the class into which the USPTO classified the patent. (This class is printed in bold on the face of the patent, and it is called the "Main Class" in the IPDD database.)

We determined the Group Art Unit to which that class belongs at this URL: *https://www.uspto.gov/patents-application-process/patent-search/understanding-patent-classifications/patent-classification*.

We used this URL to find the TC into which the patent belongs: *https://www.uspto.gov/patent/contact-patents/patent-technology-centers-management.*

Here are the TCUs that we reclassified as TC 3600-Traditional:

- TCU (3610 ("Surface Transportation"),
- TCU 3630 ("Static Structures, Supports and Furniture"),
- TCU 3640 ("Aeronautics, Agriculture, Fishing, Trapping, Vermin Destroying, Plant and Animal Husbandry, Weaponry, Nuclear Systems and License & Review"),
- TCU 3650 ("Material and Article Handling"),
- TCU 3670 ("Wells, Earth Boring/Moving/Working, Excavating, Mining, Harvesters, Bridges, Roads, Petroleum, Closures, Connections.

1. The United States Patent and Trademark Office organizes patents under various technology centers. Centers used here can be found at: *https://www.uspto.gov/patent/contact-patents/patent-technology-centers-management.*

Table A1. TC Names Used By The USPTO		
Technology Center	USPTO Name	As Referred To Or Analyzed In This Paper
1600	Biotechnology and Organic Chemistry	Biotech/Organic Chem
1700	Chemical and Materials Engineering	Chemical/Materials Engineering
2100	Computer Architecture and Software	Computer Architecture
2400	Networking, Multiplexing, Cable, and Security	Networking
2600	Communications	Communications
2800	Semiconductors/Memory, Circuits/Measuring and Testing, Optics/Photocopying, Printing/Measuring and Testing	Semiconductors
2900	Designs	Not analyzed
3600	Transportation, Construction, Electronic Commerce, Agriculture, National Security and License and Review	- 3600 E-commerce - 3600 Traditional
3700	Mechanical Engineering, Manufacturing and Medical Devices/Processes	Mech E/Med. Devs.

§ 3 Counting Office Actions

For each IPR patent, we determined the number of office actions that occurred during patent prosecution by counting the entries in the IPDD database. We counted unique entries with document codes of "CTNF" or "CTFR" for Non-Final Rejections and Final Rejections. In cases where no CTNF or CTFR entries were found, we examined the "Transaction History" field in the IPDD database for the text "Non-Final Rejection" or "Final Rejection." We counted the notice of allowance as one office action. If the petitioned patent was a Re-Issue, we calculated its office actions, as above, for it and for its parent patent and added the two together. In this case we added one for the notice of allowance but did not add one for the re-issue certificate.

§ 4 Statistical Tests

In our analyses, we used binomial tests and t-tests to analyze the effects of technology center, provenance, number of office actions and forward references on IPR institution decisions and final written decisions.

Binomial Test

In the technology center and provenance analyses, we used binomial tests to test our hypotheses. We selected the binomial test because each observation in our data has a binary outcome—*e.g.*, a given proceeding going through the IPR process is either instituted or not instituted. Binomial tests are widely used test for binary data.

The binomial test is an exact goodness-of-fit test that can be used to compare the observed distribution of binary results of a sample to an expected outcome based on a larger population.[2] For example, a binomial test can be used to test the likelihood of having three defective fuses from a sample of 20 fuses when the defective rate is five percent,[3] the likelihood of having

2. George E.P. Box, J. Stuart Hunter, and William G. Hunter, Statistics for Experimenters: "Design, Innovation, and Discovery" (2nd ed. Hoboken, NJ: *Wiley-Interscience*, 2005), 48-49; Dennis D. Wackerly, William Mendenhall III, and Richard L. Schaeffer, "Mathematical Statistics with Applications" (3rd ed. Belmont: *Duxbury Press*, 1996) 88-89; Ronald E. Walpole, Raymond H. Meyers, Sharon L. Meyers and Keying Ye, "Probability & Statistics for Engineers & Scientists" (8th ed. Upper Saddle River, NJ: *Pearson Prentice Hall*, 2007), 143-45; John H. McDonald, "Handbook of Biological Statistics" (3rd ed. Baltimore, MD: *Sparky House Publishing*, 2014).

Other potential tests we considered using were the hypergeometric test and the Z-test. The hypergeometric test is very similar to the binomial test except that it assumes trials are conducted without replacement, hence each trial has a difference success rate. Walpole *et al*., "Probability & Statistics," 152-153; Wackerly, Mendenhall, and Schaeffer, "Mathematical Statistics," 107. The hypergeometric test will be more accurate than the binomial test when the sample size is large relative to the population, but when the sample is small relative to the population, the binomial and hypergeometric tests will yield the similar results. Walpole *et al.*, Probability & Statistics, 155. Therefore, practically, the difference between binomial and hypergeometric tests is minimal, especially with big data sets.

Another possible method of analysis was conducting two-proportion Z-tests. Assuming that proceeding and patent data are randomly selected from patent population, we could test whether the institution rate and survival rate of final decision for each TC are different than other TCs. For a large sample like what we have here, the institution rate across all tech centers and the institution rate of other TCs are close.

3. Wackerly, "Mathematical Statistics," 91-92.

two of four components survive when the survival rate is 75 percent,[4] or the likelihood of a dog using his right paw 8 of 10 times when the dog is known to use his right 50 percent of the time.[5]

While approximate tests like the Pearson's chi-squared test and the G-test are available to test nominal variables, McDonald recommends that wherever possible, the exact goodness-of-fit test should be performed.[6] In general, as sample sizes get smaller, underestimation of p-values by approximate tests like the chi-squared test and the G-test becomes even more pronounced.[7] The result of underestimated p-values would be failure to reject a null hypothesis when the hypothesis would be rejected under an exact goodness-of-fit test.[8]

Null Hypotheses

Null hypotheses in Technology Center and Provenance analyses are that the institution rate or survival rate of at least one claim in a final written decision of a given technology center or ownership category are the same as the respective rates across all technology centers—*i.e.*, the overall IPR average. The alternate hypotheses are that the individual rates are different from those of the overall IPR average. Formally, for Tech Center (TC) 1600, for example, the hypotheses are:

Null: Institution Rate$_{TC1600}$

$=$Institution Rate$_{All\ TCs}$

Alternative: Institution Rate$_{TC1600}$

$\neq$Institution Rate$_{All\ TCs}$

Null: Survival Rate of Final Decision$_{TC1600}$

$=$Survival Rate of Final Decision$_{All\ TCs}$

Alternative: Survival Rate of Final Decision$_{TC1600}$

$\neq$Survival Rate of Final Decision$_{All\ TCs}$

For Provenances analysis, the hypothesis for university patents for example are:

Null: Institution Rate$_{University}$

$=$Institution Rate$_{All\ IPR}$

Alternative: Institution Rate$_{University}$

$\neq$Institution Rate$_{All\ IPR}$

Null: Survival Rate of Final Decision$_{University}$

$=$Survival Rate of Final Decision$_{All\ IPR}$

Alternative:Survival Rate of Final Decision$_{University}$

$\neq$Survival Rate of Final Decision$_{All\ IPR}$

Each test requires the assumption that a sample is to be randomly selected and sample observations are to be independent of each other. Given that the IPR process itself is not a random selection process and that patents in the IPR data set may have familial relations to each other, it is possible that the data do not fully follow the assumptions, particularly for smaller samples.

Multiple Comparisons

Because we conduct a statistical test for each TC and each provenance category, the multiple comparison issue applies to our analyses. Multiple comparison issues arise where multiple tests are performed, particularly when the number of features studied is much higher than the number of observations.[9] As described by McDonald, failing to correct for multiple comparisons can lead to increased acceptance of false positives caused by incorrectly rejecting the null hypothesis:

> [I]f you do 100 statistical tests, and for all of them the null hypothesis is actually true, you'd expect about five of the tests to be significant at the P<0.05 level, just due to chance. In that case, you'd have about five statistically significant results, all of which were false positives. The cost, in time, effort and perhaps money, could be quite high if you based important conclusions on these false positives, and it would be embarrassing for you once other people did further research and found that you'd been mistaken.[10]

Correcting for multiple comparisons is not simply a matter of minimizing false positives (type I errors). Many statisticians, including Gelman,[11] Perneger,[2]

4. Walpole *et al.*, "Probability & Statistics," 145.

5. McDonald, "Handbook of Biological Statistics," 30.

6. Ibid., 49, 56, 88-90, 93. McDonald recommends a sample size of larger than 1000 as the threshold of when to perform an approximate goodness-of-fit test over an exact goodness-of-fit test.

7. Ibid., 86-87.

8. Ibid.

9. Trevor Hastie, Robert Tibshirani , and Jerome Friedman, "The Elements of Statistical Learning: Data Mining, Inference, and Prediction," (2nd ed. New York, NY: *Springer*, 2008), 684; Hervé Abdi, "The Bonferonni and Šidák Corrections for Multiple Comparisons" (In "Encyclopedia of Measurement and Statistics," edited by Neil J. Salkind, Thousand Oaks: *Sage*, 2007), *https://www.utdallas.edu/~herve/Abdi-Bonferroni2007-pretty.pdf*; Ronald J. Feise, "Do Multiple Outcome Measures Require P-Value Adjustment?" *BMC Medical Research Methodology* 2 (2002): 8, *http://bmcmedresmethodol.biomedcentral.com/articles/10.1186/1471-2288-2-8*; John H. McDonald, "Handbook of Biological Statistics," 257-63; Joseph P. Romano, Azeem M. Shaikh, and Michael Wolf, "Multiple Testing," (The *New Palgrave Dictionary of Economics*. Online Edition. Eds. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan, 2010), *http://www.dictionaryofeconomics.com/article?id=pde2010_M000425> doi:10.1057/9780230226203.3826*; Walpole *et al*., "Probability and Statistics," 145.

10. McDonald, "Handbook of Biological Statistics," 257.

11. Andrew Gelman, Jennifer Hill, and Masanao Yajima, "Why We (Usually) Don't Have to Worry About Multiple Comparisons," *Journal of Research on Educational Effectiveness* 5 (2012): 189-211, *http://www.stat.columbia.edu/~gelman/research/published/multiple2f.pdf.*

12. Thomas V. Perneger, "What's Wrong with Bonferroni Adjustments," BMJ 316 (1998): 1236, *http://www.bmj.com/content/316/7139/1236.*

and Feise,[13] raise the concern that although multiple comparison corrections reduce the likelihood of false positives, the corrections also increase the possibility of false negatives (type II errors),, as well as the necessity to increase the sample size. Minimizing false positives at the expense of increased false negatives also increases the likelihood of rejecting a statistically insignificant factor that has a real effect. While false positives are an important concern and could result in embarrassment or additional analysis, McDonald notes "[t]he cost of a false negative, on the other hand, could be that you've missed out on a hugely important discovery."[14] Therefore, multiple comparison corrections must be approached as a question of optimization instead of a question of minimization, wherein the levels of false positives are balanced with the optimal levels of false negatives.

Benjamini-Hochberg Correction

Specifically, we used Benjamini-Hochberg correction, a correction based on the false discovery rate that allows for us to optimize false positives and false negatives compared to more restrictive methods like the *Bonferroni* method.[15] Conceptually, the false discovery rate is the expected proportion of rejected null hypotheses—*i.e.*, significant results—that were incorrectly rejected.[16] As employed by the Benjamini-Hochberg correction, the false discovery rate is an acceptable level of expected false positives chosen ex ante by the experimenter in order to balance optimized levels of false positives and false negatives.[17] For our hypothesis testing, we have employed the Benjamini-Hochberg correction using a false discovery rate of 0.10.[18]

The actual correction is performed by ordering the p-values of the results from lowest to highest and assigning them a rank i. Using the pre-selected false discovery rate Q, compare the actual p-values to the Benjamini-Hochberg critical values $(i/m)Q$, where m is the number of comparisons being made. The highest actual p-value that is less than its corresponding Benjamini-Hochberg critical value and any tests with p-values lower than that p-value are significant.[19]

T-Test

We used Welch's t-test for hypotheses in forward reference and office action analyses.[20] More specifically, we test the equality of two sample means of forward references/office actions between the instituted and not instituted petitions/patents in the institution decision, and survived and not survived petitions/patents in the final written decision.

We used paired t-tests in the analysis that compares average numbers of forward references and office actions of IPR patents verses average patents randomly sampled from the U.S. patent database. The numbers of patents sampled are matched.[21]

Z-Score

Because the number of forward references and office actions is correlated with age and technology, we corrected age and technology effects by calculating each patent's Z-score for the number of forward references and for office actions compared to its peer group.[22] A peer group is defined as the group of patents having the same age and field of search.

A Z-score is the number of standard deviation by which a data point is above the mean of the variable. In our case, a Z-score of the number of forward references of a patent shows how many standard deviations of the number of forward references of this patent is above or below the mean in its peer group. A positive Z-score means it is above average, while a negative number means it is below average. In terms of probability, the probability a Z-score is between -1 and 1 is 68 percent. There is a 95 percent probability that a Z-score will lie between -2 and 2 and a 99.7 percent probability between -3 and 3.[23]

§ 5 Forward Reference and Office Action methodologies

To count forward references we used the data stored in XML files associated with the IPPD database, and

13. Feise, "P-Value Adjustment," 5.

14. McDonald, "Handbook of Biological Statistics," 261.

15. Yoav Benjamini and Yosef Hochberg, "Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing," *Journal of the Royal Statistical Society, Series B* (Methodological) 57: 1 (1995) 289-300 *http://www.stat.purdue.edu/~doerge/BIOINFORM.D/FALL06/Benjamini%20and%20Y%20FDR.pdf*; Hastie, Elements of Statistical Learning, 686-87.

16. McDonald, "Handbook of Biological Statistics," 259.

17. Ibid., 259-60.

18. False discovery rates tend to be between 0.05 and 0.25. In terms of choosing a false discovery rate to use in the experiments, McDonald suggests that "[i]f the cost of additional experiments is low and the cost of a false negative (missing a potentially important discovery) is high, you should probably use a fairly high false discovery rate, like 0.10 or 0.20, so that you don't miss anything important." McDonald, "Handbook of Biological Statistics," 260. He further adds that a false discovery rate of 0.05 "is probably too low for many experiments." Ibid.

19. Ibid., 259-60.

20. Walpole *et al.*, Probability & Statistics, 345-347; B.L. Welch, "The Generalization of 'Student's' Problem when Several Different Population Variances are Involved," *Biometrika* 34 (Jan. 1947): 28-35.

21. Walpole *et al.*, "Probability & Statistics," 347.

22. We used logarithms for the numbers of forward references and office actions to make the data more normal, and hence to match more closely the t-test's assumption of normality.

23. George W. Snedecor and William G. Cochran, "Statistical Methods," (8th ed. Ames, IA: *Iowa, State Univ. Press*, 1989), 40-41, 465.

counted all those files' <citation> nodes having <fwdcit> tags in the relevant date range. For readers unfamiliar with statistics, here is a simple explanation of Z-scores and Z-scores of logs: for the Z-score, a value of +2 means that a subject patent's forward reference count is two standard deviations higher than the mean of its peer group, and a Z-score of -0.5 means that a subject patent's forward reference count is one-half a standard deviation lower than the mean of its peer group. The Z-score of log ("Z-log") is the Z-score of the natural log of the number of forward references or office actions in its peer group (*e.g.* Z-log definition for office actions corresponds to "Z Log OA" in the OA tables below). They are interpreted in the same way as a Z-score. The theoretical basis for the Z-score is that the underlying data are normally distributed. Taking the log can make log normally distributed data more normal. We observed that the forward reference data appeared to be log normally distributed, so we created a log transformation and computed the Z-score of the log.

We preferred using the Z-log for two reasons. First, as already stated, the log transformation is preferred over the non-transformed because using the log transformation is more appropriate for data, that like ours is not normally distributed but that appears to be approximately log normal. Transforming the data to make it more normal has the effect of increasing the ability to discriminate between lower numbers of forward references (*e.g.*, between 10 and 20) while decreasing the effect of being an extreme outlier (*e.g.*, 200 forward references). The two density plots below illustrate this—in the first plot of non-transformed forward references, the distributions are bunched up between 0 and 20 and have a long tail to the right. The second log-transformed plot shows much more dispersion on the lower end and a much shorter tail on the high end.

The second reason the Z-log is preferable has to do with the Z-score. Using the Z-score as opposed to the non-Z-score allows us to control for technology and age effects. Therefore, because we prefer the log-transformed to the non-transformed, and the Z-score to the non-Z-score, we ultimately prefer the Z-score of the log transformed data to the others.

Figure A1. Density Plot Of FR And Peer FR

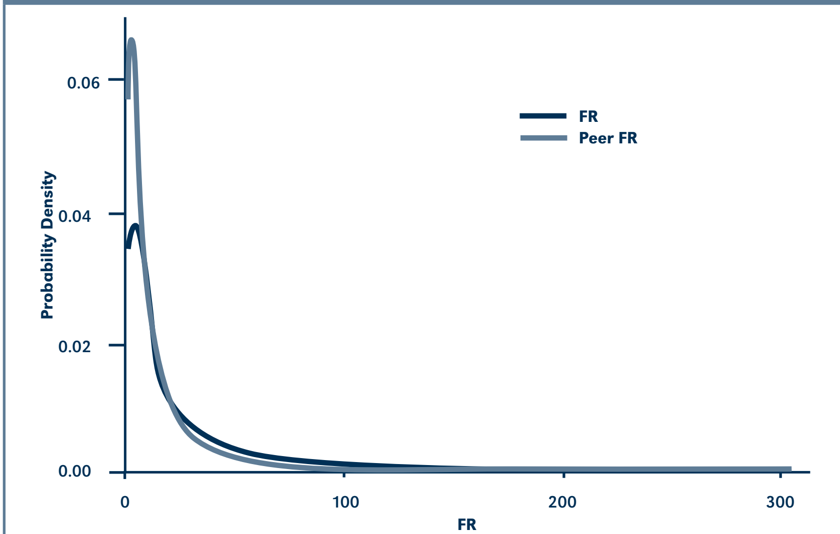
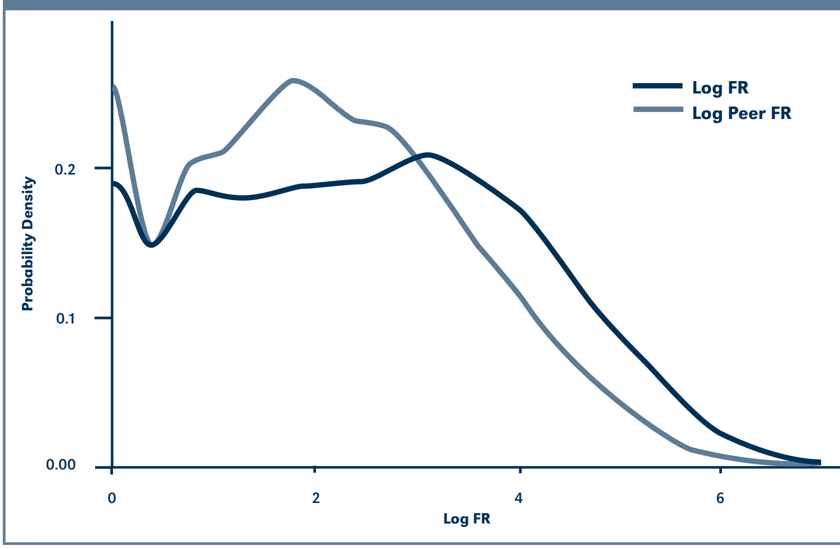


Figure A2. Density Plot Of Log FR And Peer Log FR



Figures A1 and A2 show the difference between the un-logged and the log comparisons.

§ 6—Office Actions—Comparison Between IPR and Non-IPR Patents

The following table shows comparisons of the mean number office actions, the log of office actions, the Z-score (correcting for patent technology and age) and Z-score of log (same). The comparisons were based on picking one peer patent for each patent for which a petition was filed on or before December 31, 2016. P-values, computed using paired t-tests, are $p \leq 0.01$, except for the Z-score for which $p=0.100$. Thus, with good confidence, IPR patents have significantly higher office actions than do patents as a whole.

Table A2 Summary Paired T-Tests of Office Actions

§ 7—Data for C2 and C3 Combined Analyses

Table A2. Summary Paired T-Tests Of Office Actions

	μ	σ	μ_{peer}	σ_{peer}	n	p-value
OA	2.733	1.748	2.620	2.019	3482	0.006
Log OA	0.836	0.575	0.743	0.658	3482	0.000
Z OA	0.219	1.916	0.156	1.179	3482	0.100
Z Log OA	0.245	1.946	0.147	1.108	3482	0.010

Table A3. Binomial Tests On Actual Survival Of IPR By TC (FDR=0.1)

Tech Center	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
1600	124	247	0.502	0.438	0.566	0.000	significant
1700	68	239	0.285	0.228	0.346	0.009	significant
2100	144	317	0.454	0.399	0.511	0.000	significant
2400	207	492	0.421	0.377	0.466	0.003	significant
2600	175	536	0.326	0.287	0.368	0.062	significant
2800	136	530	0.257	0.220	0.296	0.000	significant
3600EC	54	93	0.581	0.474	0.682	0.000	significant
3600Trad	90	278	0.324	0.269	0.382	0.121	
3700	117	373	0.314	0.267	0.363	0.037	significant
All TCs	1115	3105	0.359				

Figure A3. Actual Survival Rates Of IPR By TC

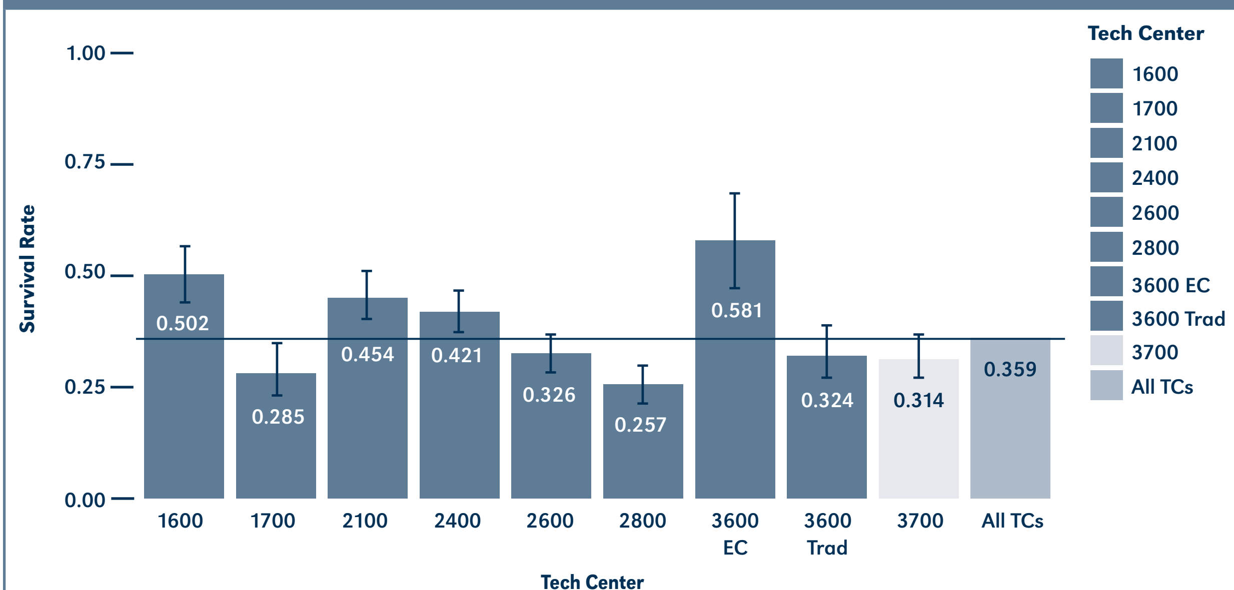
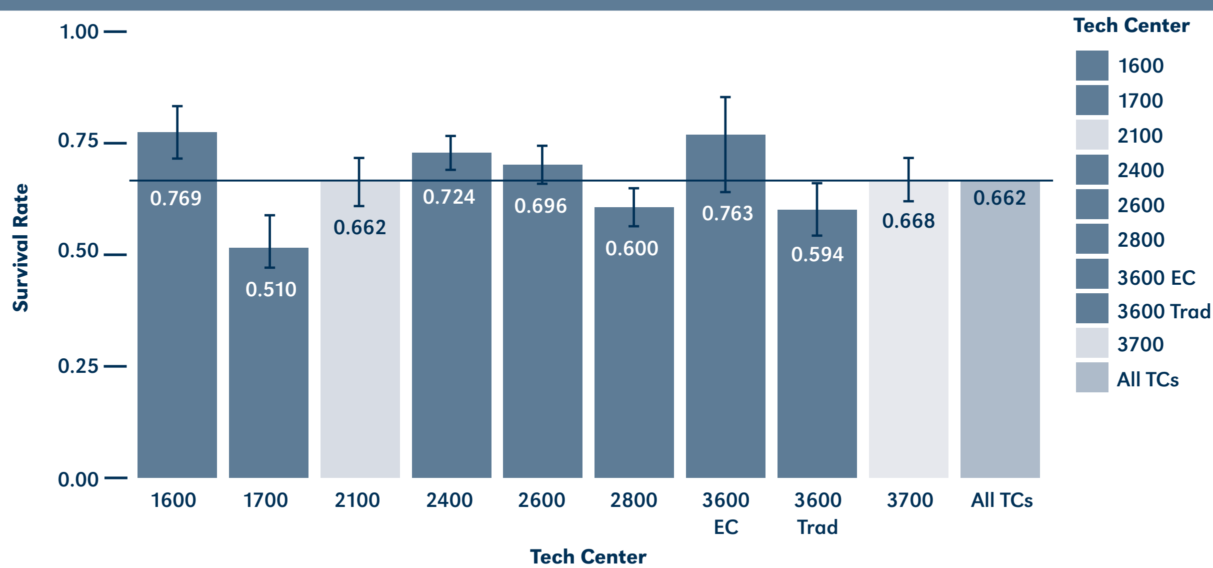


Table A4. Binomial Tests On Actual Survival Of IPR By TC (Settlement As Favorable) (FDR=0.1)							
Tech Center	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
1600	190	247	0.769	0.712	0.820	0.000	significant
1700	122	239	0.510	0.445	0.575	0.000	significant
2100	210	317	0.662	0.607	0.714	0.512	
2400	356	492	0.724	0.682	0.763	0.002	significant
2600	373	536	0.696	0.655	0.735	0.050	significant
2800	318	530	0.600	0.557	0.642	0.002	significant
3600EC	71	93	0.763	0.664	0.845	0.022	significant
3600Trad	165	278	0.594	0.533	0.652	0.011	significant
3700	249	373	0.668	0.617	0.715	0.426	
All TCs	2054	3105	0.662				

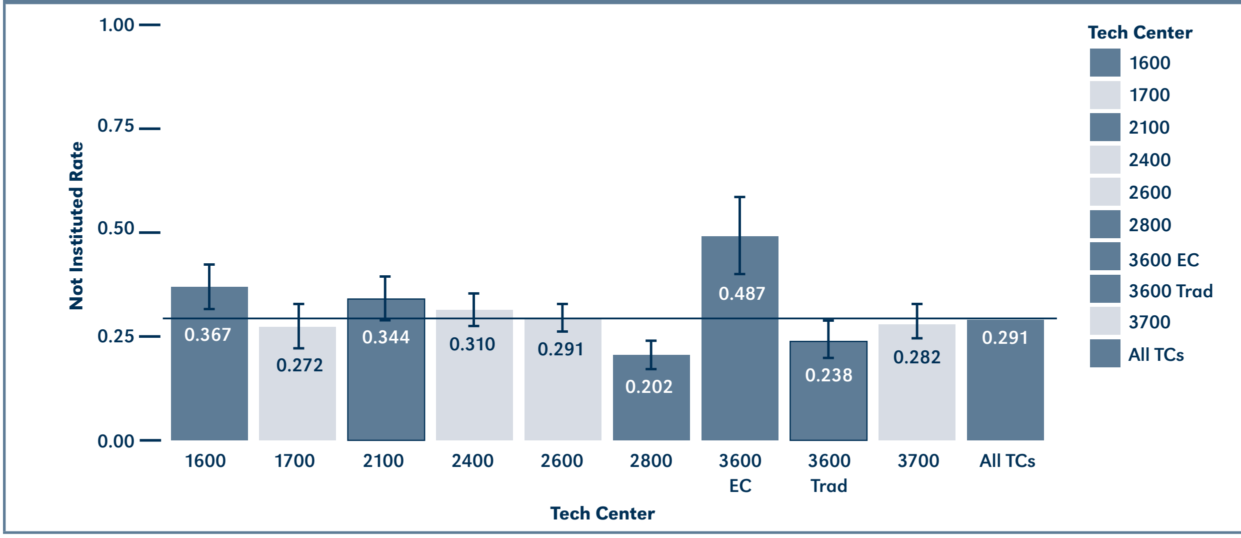
Figure A4. Actual Survival Rates Of IPR By TC (Settlement As Favorable)



§ 8—Data for *ID vs TC*

Table A5. Binomial Tests of Institution Decisions by TC (FDR = 0.1)							
Tech Center	Not Instituted	n	Not Inst. Rate	CI-lower	CI-upper	p-value	BH Significant
1600	143	390	0.367	0.319	0.417	0.001	significant
1700	77	283	0.272	0.221	0.328	0.265	
2100	133	387	0.344	0.296	0.393	0.014	significant
2400	193	622	0.310	0.274	0.348	0.154	
2600	203	697	0.291	0.258	0.327	0.507	
2800	126	625	0.202	0.171	0.235	0.000	significant
3600EC	55	113	0.487	0.392	0.583	0.000	significant
3600Trad	85	357	0.238	0.196	0.286	0.015	significant
3700	138	489	0.282	0.243	0.324	0.356	
All TCs	1153	3963	0.291				

Figure A5. Summary Of Institution Decisions By TC



§ 9—Success for Various Patent Ownership Categories

We found that the results for current assignees are generally more significant than the results for first assignees. Results for current assignees are presented below.

Table A6. Binomial Tests of Institution Decisions by CA Provenance Category (FDR=0.1)

Provenance Category	Not Instituted	n	Not Inst. Rate	CI-lower	CI-upper	p-value	BH Significant
University	20	75	0.267	0.171	0.381	0.375	
Small Entity	346	1304	0.265	0.242	0.290	0.022	significant
Other	787	2584	0.305	0.287	0.323	0.067	
All	1153	3963	0.291				

Figure A6. Summary Of Institution Decisions By CA Provenance Category

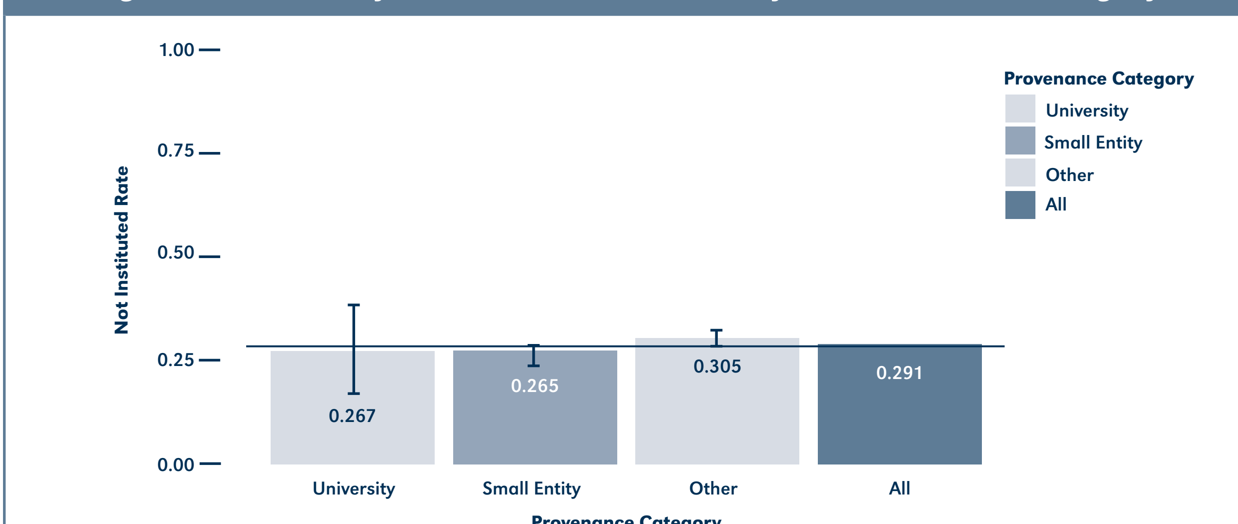


Table A7. Binomial Tests of Final Written Decisions by CA Provenance Category (All Petitions) (FDR=0.1)							
Provenance Category	Not Instituted	n	Not Inst. Rate	CI-lower	CI-upper	p-value	BH Significant
University	11	34	0.324	0.174	0.505	0.422	
Small Entity	122	525	0.232	0.197	0.271	0.001	significant
Other	276	826	0.334	0.302	0.367	0.009	significant
All	409	1385	0.295				

Figure A7. Summary of Final Written Decisions by CA Provenance Category (All Petitions)

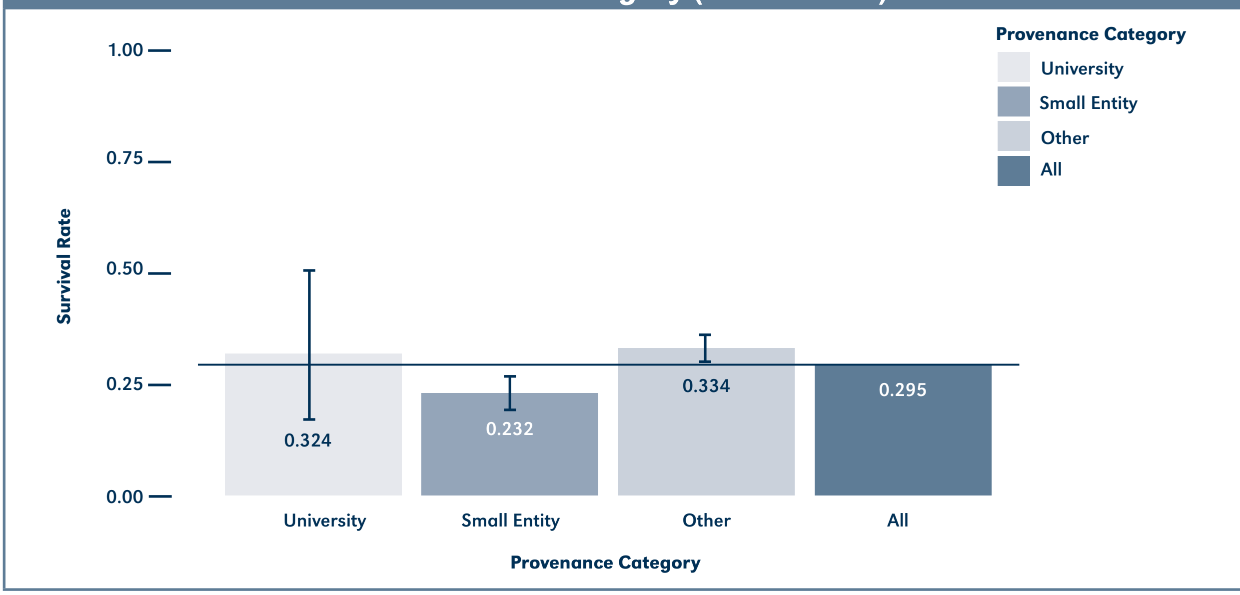


Table A8. Binomial Tests of Actual Survival Of IPR By CA Provenance Category (FDR=0.1)							
Provenance Category	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
University	26	71	0.366	0.255	0.489	0.495	
Small Entity	341	1112	0.307	0.280	0.335	0.000	significant
Other	748	1922	0.389	0.367	0.411	0.003	significant
All	1115	3105	0.359				

Figure A8. Summary of Actual Survival Of IPR By CA Provenance Category

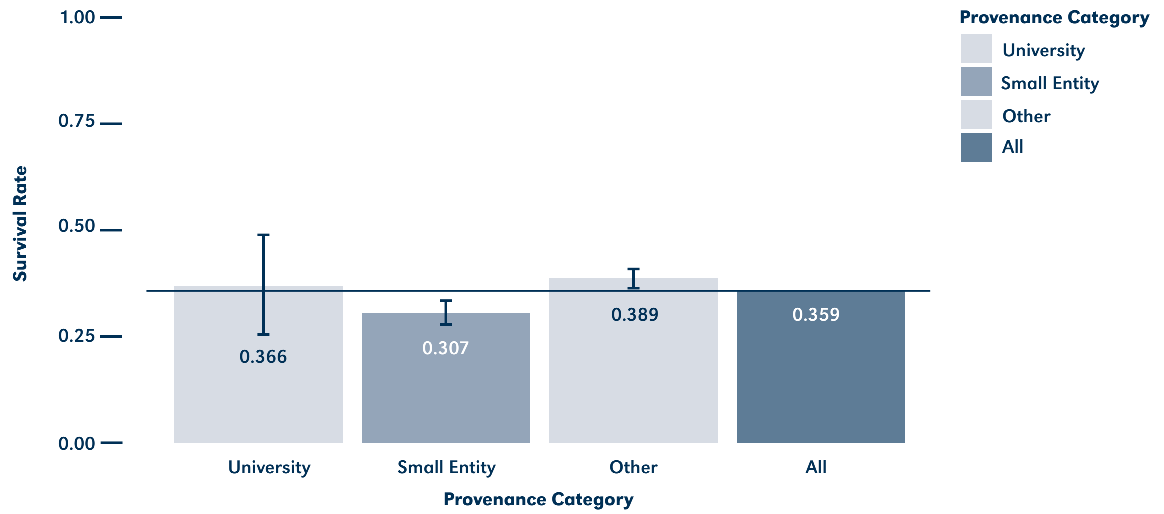


Table A9. Binomial Tests of Institution Decisions by CA Provenance Quartile (FDR=0.1)

Provenance Quartile	Not Instituted	n	Not Inst. Rate	CI-lower	CI-upper	p-value	BH Significant
1	25	74	0.338	0.232	0.457	0.221	
2	104	330	0.315	0.265	0.368	0.182	
3	323	1067	0.303	0.275	0.331	0.208	
4	701	2492	0.281	0.264	0.299	0.150	
All	1153	3963	0.291				

Figure A9. Summary Of Institution Decisions By CA Provenance Quartile

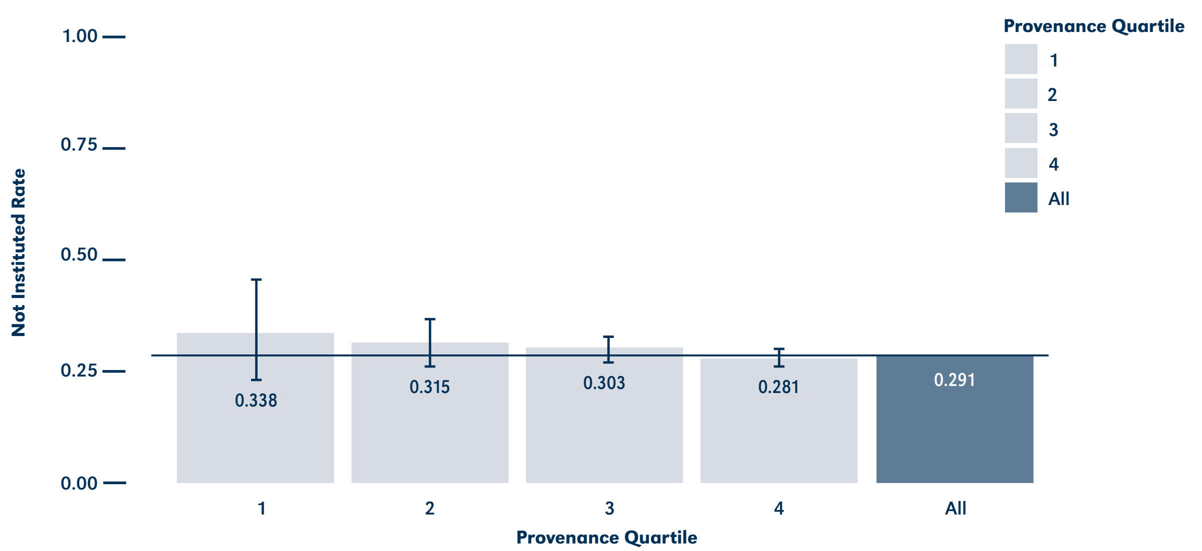


Table A10. Binomial Tests of Final Written Decisions by CA Provenance Quartile (All Petitions) (FDR=0.1)							
Provenance Quartile	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
1	5	13	0.385	0.139	0.684	0.332	
2	48	109	0.440	0.345	0.539	0.001	significant
3	115	374	0.331	0.282	0.384	0.080	
4	241	916	0.263	0.235	0.293	0.017	significant
All	409	1385	0.295				

Figure A10. Summary Of Final Written Decisions By CA Provenance Quartile (All Petitions)

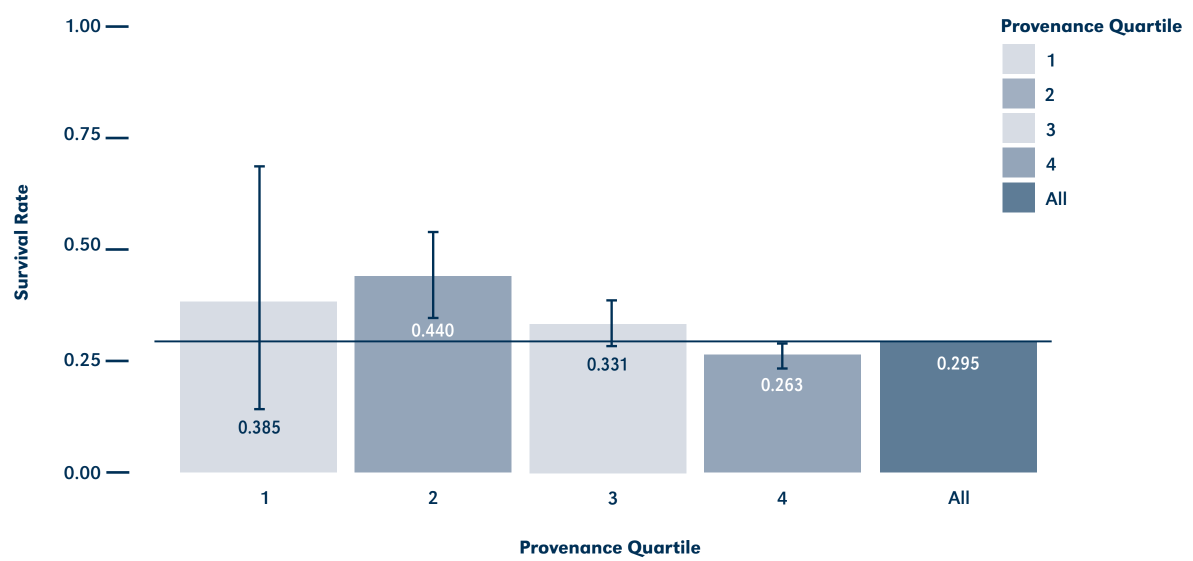
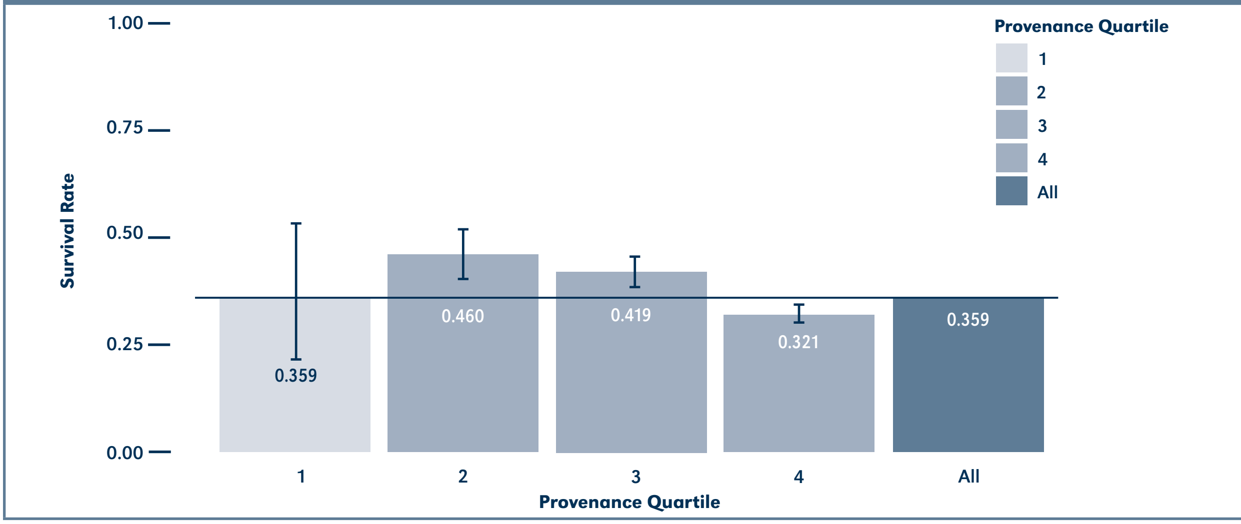


Table A11. Binomial Tests of Actual Survival of IPR by CA Provenance Quartile (FDR=0.1)							
Provenance Quartile	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
1	14	39	0.359	0.212	0.528	0.572	
2	131	285	0.460	0.401	0.519	0.000	significant
3	331	790	0.419	0.384	0.454	0.000	significant
4	639	1991	0.321	0.300	0.342	0.000	significant
All	1115	3105	0.359				

Figure A11. Summary of Actual Survival of IPR by CA Provenance Quartile



§ 10 —Data for Entity Status Changes

Table A12. Binomial Tests of Institution Decisions by Provenance Entity Status Change (FDR=0.1)

Provenance Category	Not Instituted	n	Not Inst. Rate	CI-lower	CI-upper	p-value	BH Significant
Other to Small Entity	53	247	0.215	0.165	0.271	0.004	significant
Other to University	0	1	0.000	0.000	0.975	0.709	
Small Entity to Other	24	90	0.267	0.179	0.370	0.353	
Small Entity to University	0	2	0.000	0.000	0.842	0.503	
University to Other	0	5	0.000	0.000	0.522	0.179	
University to Small Entity	0	5	0.000	0.000	0.522	0.179	
All	1153	3963	0.291				

Figure A12. Summary of Institution Decisions by Provenance Entity Status Change

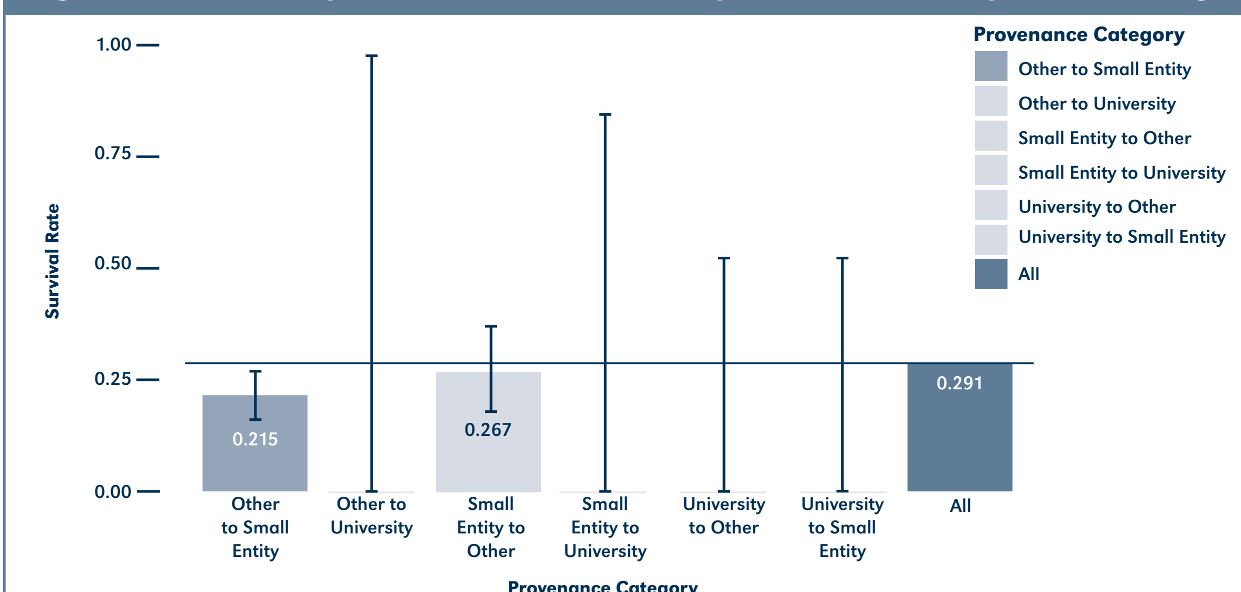


Table A13. Binomial Tests Of Final Written Decisions By Provenance Entity Status Change (All Petitions) (FDR=0.1)							
Provenance Category	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
Other to Small Entity	39	97	0.402	0.304	0.507	0.016	significant
Small Entity to Other	16	29	0.552	0.357	0.736	0.003	significant
Small Entity to University	1	2	0.500	0.013	0.987	0.503	
University to Other	2	3	0.667	0.094	0.992	0.210	
University to Small Entity	2	2	1.000	0.158	1.000	0.087	
All	409	1385	0.295				

Figure A13. Summary Of Final Written Decisions By Provenance Entity Status Change (All Petitions)

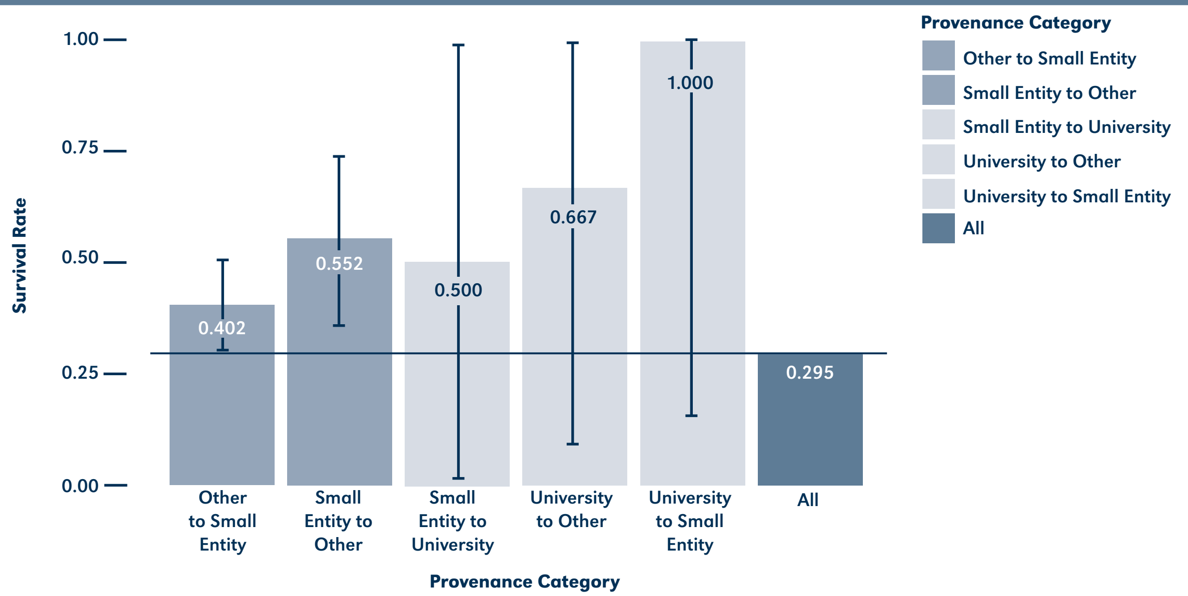
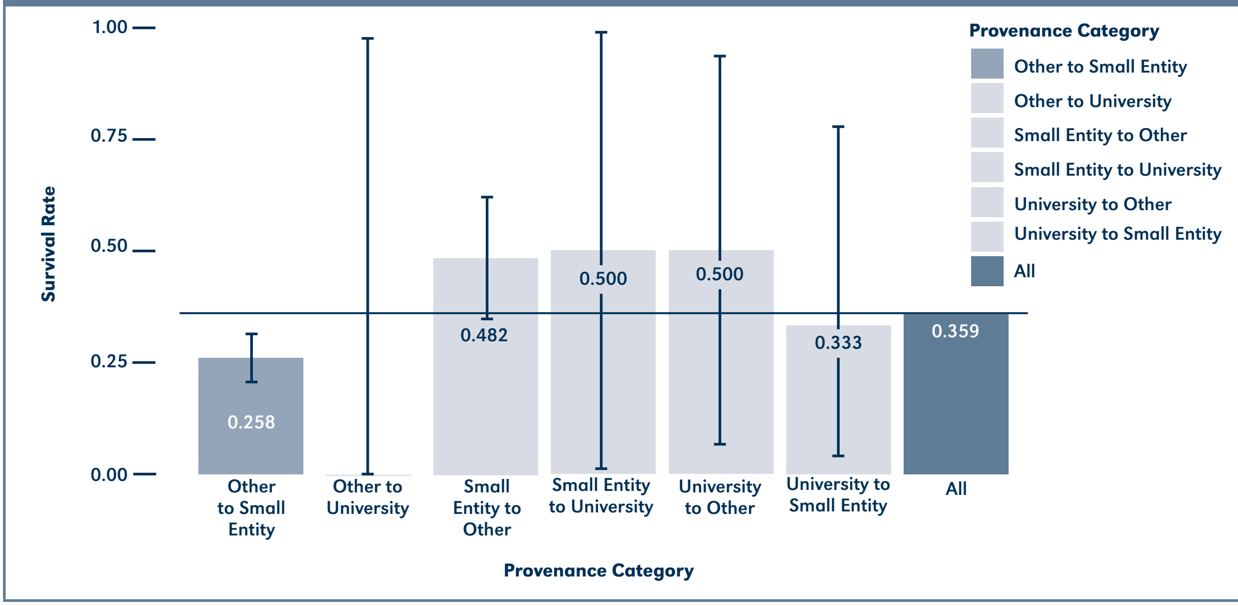


Table A14. Binomial Tests Of Actual Survival Of IPR By Provenance Entity Status Change (FDR = 0.1)							
Provenance Category	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
Other to Small Entity	70	271	0.258	0.207	0.315	0.000	significant
Other to University	0	1	0.000	0.000	0.975	0.641	
Small Entity to Other	27	56	0.482	0.347	0.620	0.039	
Small Entity to University	1	2	0.500	0.013	0.987	0.589	
University to Other	2	4	0.500	0.068	0.932	0.453	
University to Small Entity	2	6	0.333	0.043	0.777	0.629	
All	1115	3105	0.359				

Figure A14. Summary Of Actual Survival Of IPR By Provenance Entity Status Change



§ 11—Results for Other Forward Reference Measures, Self-Cites, and Examiner Citations

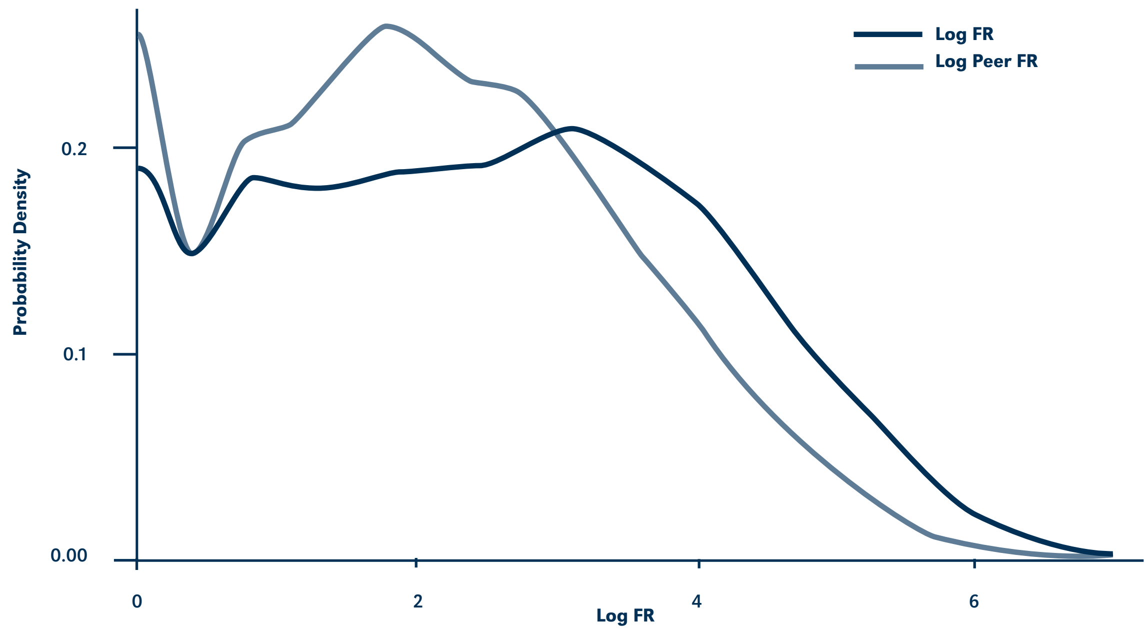
IPR patents are also statistically different from patents as a whole with respect to forward references. Table A15 shows comparisons of the mean number of forward references, the log of forward references, the Z-score (correcting for patent technology and age) and Z-log (same). The comparisons were based on picking one peer patent for each patent for which a petition was filed on or before December 31, 2016. P-values, computed using paired t-tests, are all $p < 0.001$.

Table A15. Summary of Paired T-Tests of Forward References

	μ	σ	μ_{peer}	σ_{peer}	n	p-value
FR	36.201	72.35	19.719	41.24	3482	<0.001
Log FR	2.427	1.61	2.019	1.40	3482	<0.001
Z FR	0.556	1.58	0.289	1.33	3482	<0.001
Z Log FR	0.581	1.03	0.380	1.08	3482	<0.001

Figure A15 below graphically shows the difference between log of forward reference counts for IPR patents and matched patents. IPR patents have more forward references than do patents as a whole as indicated by the fact that the IPR density curve lies slightly to the right of their peers' density curve.

Figure A15. Density Plot Of Log FR And Peer Log FR



§ 12 – Results for Office Actions Measures

Table A16. Summary Of T-Tests Of Log Of Office Actions By Outcome Variable[24]

	μ_1	σ_1	n_1	μ_2	σ_2	n_2	p-value
A	0.822	0.584	2810	0.872	0.580	1153	0.013
B1	0.753	0.588	409	0.767	0.556	976	0.691
B2	0.747	0.598	367	0.749	0.560	843	0.964
C	0.794	0.598	1115	0.755	0.564	1051	0.122

Table A17. Summary Of T-Tests Of Z Of Log Of Office Actions By Outcome Variable[25]

	μ_1	σ_1	n_1	μ_2	σ_2	n_2	p-value
A	0.290	2.05	2810	0.268	1.82	1153	0.743
B1	0.308	2.28	409	0.418	2.52	976	0.429
B2	0.337	2.38	367	0.439	2.66	843	0.510
C	0.363	2.22	1115	0.381	2.45	1051	0.860

24. Recognizing that office actions potentially suffer from normality issues, we confirmed our results using a Mann-Whitney U-test, a non-parametric test that does not require an assumption of normality in the data.

25. See *supra* note 24.

Figure A16. Histogram of Office Actions by Outcome Variable

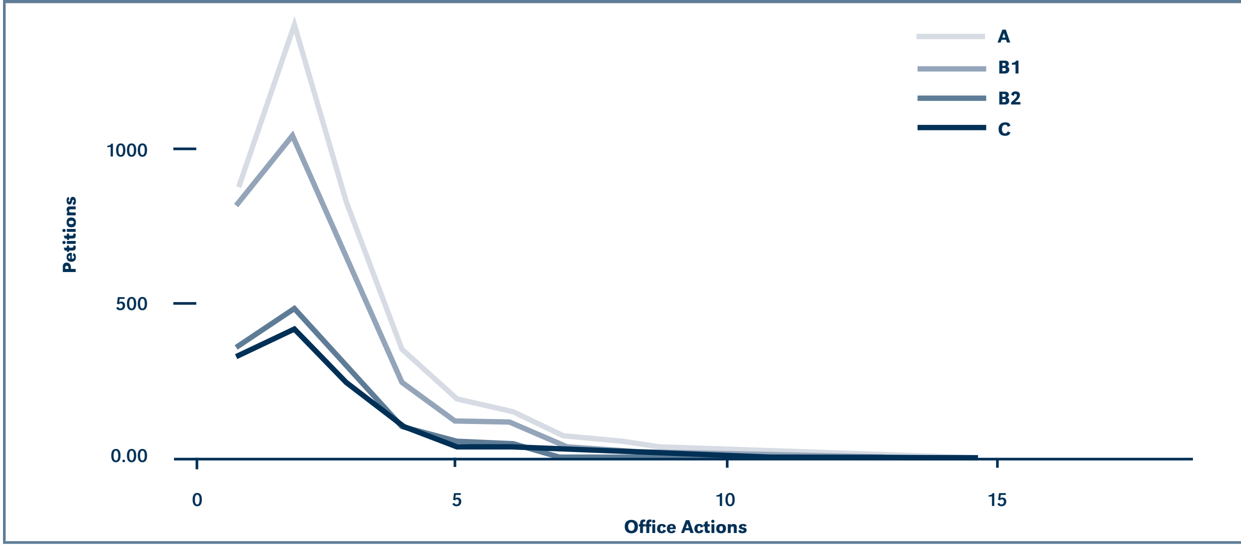


Table A1B. Binomial Tests Of Institution Decisions By Number Of Office Actions (FDR = 0.1)

Office Actions	Not Instituted	n	Not Inst. Rate	CI-lower	CI-upper	p-value	BH Significant
1	230	871	0.264	0.235	0.295	0.043	
2	397	1403	0.283	0.260	0.307	0.266	
3-4	356	1162	0.306	0.280	0.334	0.130	
5+	170	527	0.323	0.283	0.364	0.061	
All	1153	3963	0.291				

Table A2B. Binomial Tests Of Final Written Decisions By Number Of Office Actions (All Petitions) (FDR=0.1)

Office Actions	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
1	118	360	0.328	0.279	0.379	0.099	
2	124	490	0.253	0.215	0.294	0.022	Significant
3-4	125	391	0.320	0.274	0.368	0.158	
5+	42	144	0.292	0.219	0.373	0.503	
All	409	1385	0.295				

Table A3B. Binomial Tests Of Survival Of IPR By Number Of Office Actions (FDR=0.1)

Office Actions	Survived	n	Survival Rate	CI-lower	CI-upper	p-value	BH Significant
1	293	817	0.359	0.326	0.393	0.505	
2	354	1038	0.341	0.312	0.371	0.119	
3-4	333	894	0.372	0.341	0.405	0.212	
5+	135	356	0.379	0.329	0.432	0.230	
All	1115	3105	0.359				

References:

Abdi, Herve. "The Bonferonni and Šidák Corrections for Multiple Comparisons." In *Encyclopedia of Measurement and Statistics*, edited by Neil J. Salkind. Thousand Oaks: Sage, 2007. *https://www.utdallas.edu/~herve/Abdi-Bonferroni2007-pretty.pdf.*

Allison, John R., Mark A. Lemley, and David L. Schwartz. "Our Divided Patent System." *The University of Chicago Law Review* Vol. 82, No. 3 (Summer 2015):1073-1154.

Benjamini, Yoav, and Yosef Hochberg. "Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing." *Journal of the Royal Statistical Society*, Series B (Methodological) 57 (1995): 289-300 *http://www.stat.purdue.edu/~doerge/BIOINFORM.D/FALL06/Benjamini%20and%20Y%20FDR.pdf.*

Bass, Scott D., and Lukasz A. Kurgan. "Discovery of Factors Influencing Patent Value Based on Machine Learning in Patents in the Field of Nanotechnology." Scientometrics 82, No. 2 (2010): 217-241.

Black, Ken. "Business Statistics: for Contemporary Decision Making." Hoboken, NJ:*Wiley*, 2012.

Box, George E. P., J. Stuart Hunter, and William Gordon Hunter. "Statistics for Experimenters: Design, Innovation, and Discovery." 2nd ed. Hoboken, NJ: *Wiley-Interscience*, 2005.

Coppock, Alexander. "10 Things You Need to Know About Multiple Comparisons." *Egap.org*. (Feb. 8, 2015). *http://egap.org/methods-guides/10-things-you-need-know-about-multiple-comparisons.*

Feise, Ronald J. "Do Multiple Outcome Measures Require P-Value Adjustment?" *BMC Medical Research Methodology* 2 (2002): 8-8. *http://bmcmedresmethodol.biomedcentral.com/articles/10.1186/1471-2288-2-8.*

Gelman, Andrew, Jennifer Hill, and Masanao Yajima. "Why We (Usually) Don't Have to Worry About Multiple Comparisons." *Journal of Research on Educational Effectiveness* 5 (2012):189-211. *http://www.stat.columbia.edu/~gelman/research/published/multiple2f.pdf.*

Hastie, Trevor, Robert Tibshirani, and Jerome H. Friedman. "The Elements of Statistical Learning: Data Mining, Inference, and Prediction." 2nd ed. New York, NY: *Springer*, 2008.

Lee, Jungpyo and So Young Sohn. "What Makes the First Forward Citation of Patent Occur Earlier?" *Scientometrics*. 2017: 1588. *https://doi.org/10.1007/s11192-017-2480-1.*

McDonald, John H. "Handbook of Biological Statistics." 3rd ed. Baltimore, MD: *Sparky House Publishing*, 2014. *http://www.biostathandbook.com/HandbookBioStatThird.pdf.*

Perneger, T. V. "What's Wrong with Bonferroni Adjustments." *BMJ* 316 (1998): 1236-238. doi:10.1136/bmj.316.7139.1236. *http://www.bmj.com/content/316/7139/1236.*

Romano, Joseph P., Azeem M. Shaikh, and Michael Wolf. "Multiple Testing." *The New Palgrave Dictionary of Economics*. Online Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan. 2010. *http://www.dictionaryofeconomics.com/article?id=pde2010_M000425> doi:10.1057/9780230226203.3826*

Seltman, Howard J. "Experimental Design and Analysis." Pittsburgh, 2008. *http://www.stat.cmu.edu/~hseltman/309/Book/Book.pdf.*

Snedecor, George W., and William G. Cochran. "Statistical Methods." 8th ed. Ames, IA: *Iowa State Univ. Press*, 1989.

Wackerly, Dennis D., William Mendenhall, and Richard L. Schaeffer. "Mathematical Statistics with Applications." 3rd ed. Belmont: *Duxbury Press*, 1996.

Walpole, Ronald E., Ronald H. Meyers, Sharon L., Meyers, and Keying Ye. "Probability & Statistics for Engineers & Scientists." 8th ed. Upper Saddle River, NJ: *Pearson Prentice Hall*, 2007.

Welch, B.L. "The Generalization of 'Student's' Problem when Several Different Population Variances are Involved." *Biometrika* 34 (Jan. 1947): 28-35.